

**TIME: 180 MINUTES**  
**MAXIMUM MARKS : 240**

GENERAL INSTRUCTIONS

1. This question booklet has Three subject **PARTS**: Physics, Chemistry and Mathematics
  - A student **must answer** at least a few questions from **ALL three subject PARTS** to score minimum marks in all three subjects
  - In each subject PART there are three sections:
    - SECTION 1: Multiple Choice Question ( MCQ)
    - SECTION 2: SINGLE DIGIT NON-NEGATIVE INTEGER(SDI)
    - SECTION 3: Paragraph based MCQ (**PBQ**).
2. Read carefully instructions given in each section of the question paper.

**Part -1: Physics**  
**SECTION 1: Multiple Choice Question ( MCQ)**

- This section contains **EIGHT (8)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.
- For each question, darken the bubble corresponding to the correct answer.
- Answer to each question will be evaluated according to the following marking scheme:

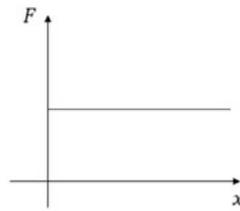
*Full Marks* : +4, If **ONLY** the bubble corresponding to the correct option is darkened.

*Zero Marks* : 0, If none of the bubble is darkened (i.e., the question is unanswered).

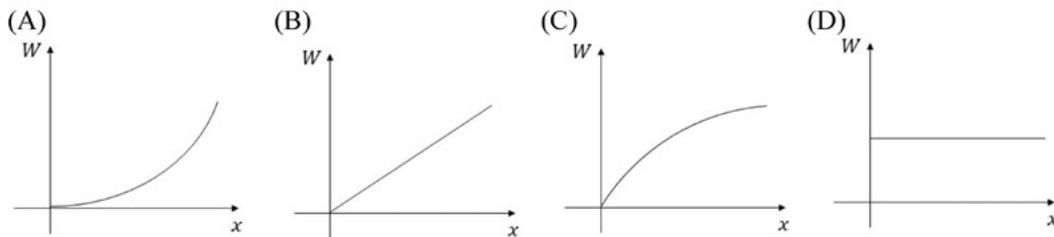
*Negative Marks* : -1, In all other cases.

- Q.1 *A spacecraft  $S_1$  is moving at a speed of  $0.8c$  ( $c$  is the speed of light) with respect to another spacecraft  $S_2$ , which is at rest. After  $t_1$  hours have passed in spacecraft  $S_1$ , the number of hours passed in spacecraft  $S_2$  is \_\_\_\_.*
- (A)  $1.7 t_1$                       (B)  $0.3 t_1$                       (C)  $t_1$                               (D)  $0.6 t_1$
- Q.2 *A radioactive decay process is given by  ${}^{238}_{92}\text{U} \rightarrow {}^{234}_{90}\text{Th} + X$ . Identify  $X$ .*
- (A) Electron                      (B) Proton                              (C) Neutron                              (D) Alpha particle
- Q.3 *The SI unit of the electromotive force (EMF) is \_\_\_\_.*
- (A) Newton (N)                      (B) Joule (J)                              (C) Volt (V)                              (D)  $\text{meter}/\text{sec}^2(\text{m}/\text{s}^2)$

Q.4 A particle is displaced by a distance  $x$  under the influence of a force  $F$  as per the following force-displacement graph:

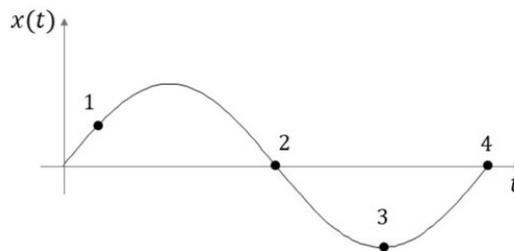


The work done ( $W$ ) by the force  $F$  as a function of displacement  $x$  can be represented as:



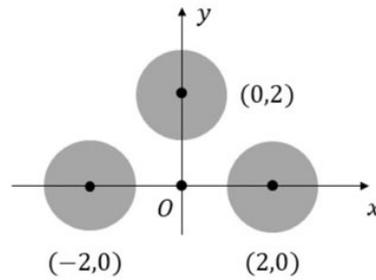
Q.5 A particle undergoes a sinusoidal motion  $x(t) = A \sin(\omega t)$  under the influence of a force  $F(t)$  as shown in the figure below. At which of the points, marked as 1, 2, 3, 4 in the figure below, the magnitude of the force  $F(t)$  will be the maximum?

(Here,  $A$  is the amplitude,  $\omega$  is the constant angular frequency, and  $t$  is the time)



- (A) 1                      (B) 2                      (C) 3                      (D) 4

- Q.6 Three spheres, each of mass  $M$  and radius 1 unit, are placed in a plane as shown in the figure below:



The direction of the gravitational force on a point particle of unit mass placed at the origin  $O$  is \_\_\_\_\_.  
(Here  $\hat{x}$ ,  $\hat{y}$  are unit vectors along the  $x$ - and  $y$ -directions, respectively)

- (A)  $\hat{x}$                       (B)  $\hat{y}$                       (C)  $-\hat{x}$                       (D)  $-\hat{y}$
- Q.7 Consider a simple pendulum where a mass  $m$  is attached to a massless, unstretchable string with one end fixed. Which of the following changes will result in the doubling of its time period?

- (I) The string length increases four times.  
(II) The acceleration due to gravity increases four times.  
(III) The string length is halved.  
(IV) The acceleration due to gravity becomes one fourth.

- (A) (I) and (IV)              (B) (I) and (II)              (C) (II) and (III)              (D) (III) and (IV)
- Q.8 Two ideal monatomic gasses,  $X$  and  $Y$ , of densities  $\rho_X = \rho$  and  $\rho_Y = 3\rho$ , respectively, are maintained at the same temperature and pressure. If the speeds of sound in gases  $X$  and  $Y$  are  $v_X$  and  $v_Y$ , respectively, then the ratio  $v_X : v_Y$  is \_\_\_\_\_.

- (A) 1:3                      (B)  $1:\sqrt{3}$                       (C) 3:1                      (D)  $\sqrt{3}:1$

<b>Part -1: Physics</b>	
<b>SECTION 2: SINGLE DIGIT NON-NEGATIVE INTEGER (SDI)</b>	
<ul style="list-style-type: none"> <li>• This section contains <b>Six (6)</b> questions.</li> <li>• The answer to each question is a <b>SINGLE DIGIT NON-NEGATIVE INTEGER i.e. 0 to 9</b>.</li> <li>• For each question, darken the bubble corresponding to the correct answer.</li> </ul>	
<ul style="list-style-type: none"> <li>• Answer to each question will be evaluated according to the following marking scheme:</li> </ul> <p><i>Full Marks</i> : +4, If <b>ONLY</b> the bubble corresponding to the correct answer is darkened.</p> <p><i>Zero Marks</i> : 0, In all other cases.</p>	

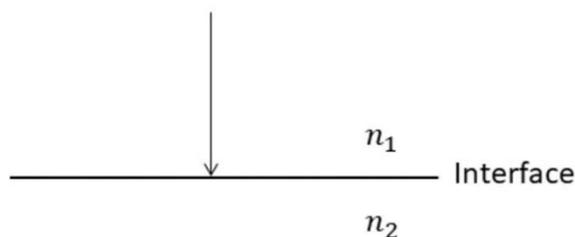
Q.9 Two radioactive elements  $R_1$  and  $R_2$  with half-lives of  $2\lambda$  and  $\lambda$ , decay to  $\frac{1}{3}$  of their initial numbers of nuclei at times  $t_1$  and  $t_2$ , respectively. The ratio  $\frac{t_1}{t_2}$  is \_\_\_\_\_.

Q.10 Consider a X-Y plane with a uniform magnetic field in the positive Y-direction. A charged particle of charge  $+q$  is at rest at the origin. Then, the magnitude of the magnetic force on the charged particle is \_\_\_\_\_ Newton.

Q.11 A uniform magnetic field of magnitude  $B = 0.04$  Tesla passes perpendicularly through a circular loop of area  $100 \text{ meter}^2$ . The magnetic flux through the loop is \_\_\_\_\_ Tesla-meter<sup>2</sup>.

Q.12 Unpolarized light of intensity  $2 \text{ Watt/meter}^2$  is incident on an ideal polarizer. The intensity of the light emerging from the polarizer is \_\_\_\_\_ Watt/meter<sup>2</sup>.

Q.13 A ray of light is incident normally at the interface separating two media of refractive indices  $n_1$  and  $n_2$ , as shown in the figure below.



The angle between the reflected ray and the transmitted ray is \_\_\_\_\_  $\pi$  Radians.

Q.14 A metallic sphere of density  $\rho$  weighs  $10$  Newton in vacuum. The weight of the sphere when completely submerged in a liquid of density  $\frac{\rho}{2}$  is \_\_\_\_\_ Newton.

**Part -1: Physics**  
**SECTION 3 : Paragraph based MCQ (PBQ)**

- This section contains **THREE (03)** paragraphs.
- Based on each paragraph, there are **TWO (02)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.
- If the answer is a numerical value with more than two decimal places, **truncate/round-off** the value to **TWO** decimal places.

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*Full Marks* : +4, If **ONLY** the bubble corresponding to the correct answer is darkened.

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Q.15 *PARAGRAPH I*

*An electron in a Hydrogen atom has a de Broglie wavelength  $\lambda$ . This system can be considered to be analogous to the vibrations of a wire loop with the nucleus fixed at its center. In Bohr's model, an electron can circle around the nucleus only if the de Broglie wavelength is  $n\lambda$ , where  $n$  is an integer.*

*Calculate the radius of the orbit when  $n = 1$  and de Broglie wavelength is 3.3 Angstrom. (Given  $\pi = 3.14$ , 1 Angstrom =  $10^{-10}$  meters)*

- (A) 0.53 Angstrom    (B) 2.51 Angstrom    (C) 1.25 Angstrom    (D) 3.14 Angstrom

Q.16 *PARAGRAPH I*

*An electron in a Hydrogen atom has a de Broglie wavelength  $\lambda$ . This system can be considered to be analogous to the vibrations of a wire loop with the nucleus fixed at its center. In Bohr's model, an electron can circle around the nucleus only if the de Broglie wavelength is  $n\lambda$ , where  $n$  is an integer.*

*The potential energy of the electron in the Hydrogen atom is  $-\frac{e^2}{4\pi\epsilon_0 r}$ , where  $e$  is the charge of the electron,  $r$  is the Bohr radius and  $\epsilon_0$  is the permittivity of free space. Then, the total energy of the electron is \_\_\_\_.*

- (A)  $-\frac{e^2}{4\pi\epsilon_0 r}$     (B)  $\frac{e^2}{4\pi\epsilon_0 r}$     (C)  $-\frac{e^2}{8\pi\epsilon_0 r}$     (D)  $\frac{e^2}{8\pi\epsilon_0 r}$

Q.17 **PARAGRAPH II**

A particle of charge  $q$  moving with velocity  $\vec{v}$ , experiences a force  $q(\vec{v} \times \vec{B})$  in a magnetic field  $\vec{B}$ .

A positively charged particle is moving in a magnetic field oriented in the positive  $y$ -direction. If at a given instant the velocity is in the positive  $x$ -direction, then the magnetic force experienced by the particle at that instant is \_\_\_\_\_.

- (A) in the positive  $x$ -direction
- (B) in the positive  $z$ -direction
- (C) in the negative  $z$ -direction
- (D) in the negative  $x$ -direction

Q.18 **PARAGRAPH II**

A particle of charge  $q$  moving with velocity  $\vec{v}$ , experiences a force  $q(\vec{v} \times \vec{B})$  in a magnetic field  $\vec{B}$ .

A charged particle of mass 0.003 kilograms performs a uniform circular motion with a linear speed  $v = 1$  meter/sec in the  $X$ - $Y$  plane in a uniform magnetic field  $\vec{B} = 10^{-3} \hat{z}$  Tesla, where  $\hat{z}$  is the unit vector along  $z$ -direction. Neglecting gravity, given that the magnitude of centripetal force is  $qvB$  and the radius of the circular trajectory is 1 meter, the magnitude of the charge in Coulomb is \_\_\_\_\_.

- (A) 1
- (B) 2
- (C) 3
- (D) 4

Q.19 **PARAGRAPH III**

The current  $I(t) = 2 \sin(\omega t)$  Ampere is flowing with a constant angular frequency  $\omega$ , through a resistor of resistance 1 ohm. This results in the energy dissipation through the resistor in the form of heat. (Here  $t$  is the time.)

The voltage drop across the resistor \_\_\_\_\_.

- (A) is in phase with the current
- (B) leads the current by a phase of  $\frac{\pi}{2}$
- (C) lags behind the current by a phase of  $\frac{\pi}{2}$
- (D) is constant in time

Q.20      *PARAGRAPH III*

*The current  $I(t) = 2 \sin(\omega t)$  Ampere is flowing with a constant angular frequency  $\omega$ , through a resistor of resistance 1 ohm. This results in the energy dissipation through the resistor in the form of heat. (Here  $t$  is the time.)*

*The rate of energy dissipation at the instant  $t = \frac{2\pi}{\omega}$  sec is \_\_\_\_ Watt.*

(A) 3

(B) 2

(C) 1

(D) 0

**Part -2: Chemistry**  
**SECTION 1: Multiple Choice Question ( MCQ)**

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Q.21 *The reaction that is accompanied by an increase in entropy is*

- (A)  $N_2(g) + 3H_2(g) \rightarrow 2NH_3(g)$
- (B)  $2H_2(g) + O_2(g) \rightarrow 2H_2O(g)$
- (C)  $CaCO_3(s) \rightarrow CaO(s) + CO_2(g)$
- (D)  $3C_2H_2(g) \rightarrow C_6H_6(g)$

Q.22 *The heat absorbed/released by the system is zero for an*

- (A) isothermal process
- (B) adiabatic process
- (C) isochoric process
- (D) isobaric process

Q.23 *The unit of rate constant for a first order reaction is*

- (A)  $mol L^{-1} s^{-1}$
- (B)  $mol^{-1} L s^{-1}$
- (C)  $mol^{-2} L^2 s^{-1}$
- (D)  $s^{-1}$

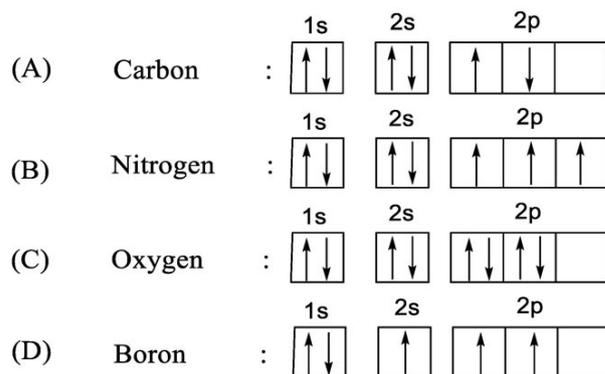
Q.24 *The INCORRECT statement regarding accuracy and precision, is*

- (A) Repeatability of measurements is called precision.
- (B) Correctness of measurements is called accuracy.
- (C) If a measurement is precise, then it is also accurate.
- (D) To get good precision, a scientist tries to repeat the measurement in exactly the same way each time.

Q.25 *The process which results in an increase in atomic number is*

- (A) gamma emission.
- (B) positron emission.
- (C) beta emission.
- (D) alpha emission.

Q.26 Among the ground state electronic configuration of elements, the correct one is



Q.27 If surface tension of water at 25 °C is 72.0 mN m<sup>-1</sup>, then its surface tension at 90 °C will be

- (A) lower than 72.0 mN m<sup>-1</sup>.                      (B) higher than 72.0 mN m<sup>-1</sup>.  
 (C) equal to 72.0 mN m<sup>-1</sup>.                        (D) equal to zero.

Q.28 Among the following amines, the one that is NOT a tertiary amine is

- (A) propane-2-amine.                                (B) trimethylamine.  
 (C) *N,N*-diethylisopropylamine.                (D) *N,N*-dimethylethylamine.

<b>Part -2: Chemistry</b>	
<b>SECTION 2: SINGLE DIGIT NON-NEGATIVE INTEGER (SDI)</b>	
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- Q.29 *The enthalpy of sublimation and enthalpy of fusion of an element X(s) are  $10 \text{ kJ mol}^{-1}$  and  $3 \text{ kJ mol}^{-1}$ , respectively. The enthalpy of vaporization (in  $\text{kJ mol}^{-1}$ ) of X(l) is \_\_\_\_\_.*
- Q.30 *At a given temperature, the ratio of root mean square speeds of gaseous  $\text{H}_2$  and  $\text{O}_2$ ,  $\frac{v_{\text{rms}}(\text{H}_2)}{v_{\text{rms}}(\text{O}_2)}$ , is \_\_\_\_\_. [Molar masses (in  $\text{g mol}^{-1}$ ):  $\text{H}_2 = 2$ ,  $\text{O}_2 = 32$ ]*
- Q.31 *For a reaction, the plot of  $\ln k$  versus  $1/T$  yields slope equals to  $-4$  (in K). If the activation energy (in  $\text{J mol}^{-1}$ ) of the reaction is  $x \times R$ , the value of  $x$  is \_\_\_\_\_. (R is the universal gas constant in  $\text{J K}^{-1} \text{ mol}^{-1}$ )*
- Q.32 *There are \_\_\_\_\_ significant figures in 0.06030*
- Q.33 *The coefficient of the permanganate ion when the following equation is balanced, is \_\_\_\_\_.*  

$$\text{MnO}_4^- + \text{Br}^- + \text{H}^+ \rightarrow \text{Mn}^{2+} + \text{Br}_2 + \text{H}_2\text{O}$$
- Q.34 *For the reaction  $2\text{C}_2\text{H}_6(\text{g}) + 7\text{O}_2(\text{g}) \rightarrow 4\text{CO}_2(\text{g}) + 6\text{H}_2\text{O}(\text{l})$  at temperature T, the difference between change in the enthalpy ( $\Delta H$ ) and change in the internal energy ( $\Delta U$ ), " $\Delta H - \Delta U$ ", is  $-x \times RT$ . The value of  $x$  is \_\_\_\_\_. (R is the universal gas constant and gases can be considered to be ideal)*

**Part -2: Chemistry**  
**SECTION 3 : Paragraph based MCQ (PBQ)**

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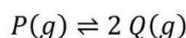
*Zero Marks* : 0, If none of the bubble is darkened (i.e., the question is unanswered).

*Negative Marks* : -1, In all other cases.

Q.35

PARAGRAPH IV

*4 moles of P undergo reaction to give 4 moles of Q at equilibrium in a 1 L container according to the reaction (no Q is present in the beginning of the reaction)*



*The equilibrium constant ( $K_c$ ) for the reaction is*

(A) 2

(B) 4

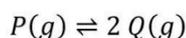
(C) 8

(D) 16

Q.36

PARAGRAPH IV

*4 moles of P undergo reaction to give 4 moles of Q at equilibrium in a 1 L container according to the reaction (no Q is present in the beginning of the reaction)*



*If the rate of consumption of P is  $1 \text{ mol L}^{-1} \text{ s}^{-1}$ , the rate of formation (in  $\text{mol L}^{-1} \text{ s}^{-1}$ ) of Q is*

(A) 1

(B) 2

(C) 3

(D) 4

Q.37

## PARAGRAPH V

The solubility product constant,  $K_{sp}$ , of a sparingly soluble halide salt  $MX$  is  $1 \times 10^{-10}$  at 300 K.

The solubility (in  $\text{mol L}^{-1}$ ) of the salt  $MX$  at 300 K is

- (A)  $1 \times 10^{-5}$  (B)  $2 \times 10^{-5}$   
(C)  $1 \times 10^{-10}$  (D)  $2 \times 10^{-10}$

Q.38

## PARAGRAPH V

The solubility product constant,  $K_{sp}$ , of a sparingly soluble halide salt  $MX$  is  $1 \times 10^{-10}$  at 300 K.

At 300 K, the solubility (in  $\text{mol L}^{-1}$ ) of the salt  $MX$  in the presence of 0.1 M  $NaX$  is

- (A)  $1 \times 10^{-5}$  (B)  $1 \times 10^{-6}$   
(C)  $1 \times 10^{-8}$  (D)  $1 \times 10^{-9}$

Q.39

## PARAGRAPH VI

Titration between oxalic acid dihydrate (a diprotic acid, hereafter referred to as OA) and sodium hydroxide is a typical acid-base titration. Phenolphthalein is used as an indicator. The appearance of a permanent light pink color indicates the end point. This titration method helps in determining the unknown concentration of sodium hydroxide using a standard OA solution. (Molar mass of OA =  $126 \text{ g mol}^{-1}$ )

The amount (in g) of OA required to make 0.5 L of 0.1 M solution is

- (A) 126 (B) 12.6 (C) 63.0 (D) 6.30

Q.40

## PARAGRAPH VI

Titration between oxalic acid dihydrate (a diprotic acid, hereafter referred to as OA) and sodium hydroxide is a typical acid-base titration. Phenolphthalein is used as an indicator. The appearance of a permanent light pink color indicates the end point. This titration method helps in determining the unknown concentration of sodium hydroxide using a standard OA solution. (Molar mass of OA =  $126 \text{ g mol}^{-1}$ )

A titration of 20 mL of 0.1 M solution of OA consumed 16 mL of sodium hydroxide solution. The concentration (in M) of this sodium hydroxide solution is

- (A) 0.250 (B) 0.125 (C) 0.0625 (D) 0.160

**Part -3: Mathematics**  
**SECTION 1: Multiple Choice Question ( MCQ)**

- This section contains **EIGHT (8)** questions.
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- For each question, darken the bubble corresponding to the correct answer.
- Answer to each question will be evaluated according to the following marking scheme:

*Full Marks* : +4, If **ONLY** the bubble corresponding to the correct option is darkened.

*Zero Marks* : 0, If none of the bubble is darkened (i.e., the question is unanswered).

*Negative Marks* : -1, In all other cases.

Q.41 The value of the sum

$$\sum_{n=1}^{2024} \frac{1}{\sqrt{n+1} + \sqrt{n}}$$

is

- (A) 24                      (B) 45                      (C) 20                      (D) 44

Q.42 For which one of the following values of  $k$ , the equation

$$\cos\left(\frac{k\pi}{7}\right) = \cos\left(\frac{6\pi}{7}\right)$$

is satisfied?

- (A) 9                      (B) 5                      (C) 8                      (D) 4

Q.43 For which one of the following values of  $p$ , does the following inequality hold for all  $x \geq 0$  ?

$$(1+x)^p \leq 1+x^p$$

- (A)  $p = \frac{3}{2}$                       (B)  $p = \frac{1}{2}$                       (C)  $p = 2$                       (D)  $p = 4$

- Q.44 Let  $f: \mathbb{R} \rightarrow \mathbb{R}$  be a two times differentiable function such that  $f''(x) \geq 0$  for every  $x \in \mathbb{R}$  and  $f'(0) > 0$ , where  $f'$  and  $f''$  denote the first and the second order derivatives of  $f$ , respectively. If  $g(x) = f(e^x)$ , then
- (A)  $g''(x) > 0$  for all  $x \in \mathbb{R}$ .  
(B)  $g''(x) = 0$  for some  $x \in \mathbb{R}$ .  
(C)  $g''(x) < 0$  for all  $x \in \mathbb{R}$ .  
(D) There exist  $x_1$  and  $x_2$  in  $\mathbb{R}$  such that  $g''(x_1) < 0$  and  $g''(x_2) > 0$ .
- Q.45 The number of possible  $3 \times 3$  matrices with entries from the set  $\{0, 1\}$  is
- (A) 8                      (B) 18                      (C) 512                      (D) 81
- Q.46 The value of  $\theta$  in the interval  $\left[0, \frac{\pi}{2}\right]$  satisfying  $1 + \cos \theta + \cos^2 \theta + \cos^3 \theta + \dots = 4 + 2\sqrt{3}$  is
- (A)  $\frac{\pi}{2}$                       (B)  $\frac{\pi}{3}$                       (C)  $\frac{\pi}{4}$                       (D)  $\frac{\pi}{6}$
- Q.47 The number of one-one (injective) functions from the set  $A = \{-1, 0, 1\}$  to the set  $B = \{1, 2, 3, 4\}$  is
- (A) 12                      (B) 81                      (C) 64                      (D) 24
- Q.48 A heap of sand is in the form of a cone whose height is 9 meters and it contains  $432\pi$  cubic meters of sand. The minimum area, in square meters, of the canvas required to cover the heap is
- (A)  $12\sqrt{63}\pi$                       (B)  $324\pi$                       (C)  $180\pi$                       (D)  $192\pi$

<b>Part -3: Mathematics</b>	
<b>SECTION 2: SINGLE DIGIT NON-NEGATIVE INTEGER (SDI)</b>	
<ul style="list-style-type: none"> <li>• This section contains <b>Six (6)</b> questions.</li> <li>• The answer to each question is a <b>SINGLE DIGIT NON-NEGATIVE INTEGER i.e. 0 to 9.</b></li> <li>• For each question, darken the bubble corresponding to the correct answer.</li> </ul>	
<ul style="list-style-type: none"> <li>• Answer to each question will be evaluated according to the following marking scheme:</li> </ul> <p><i>Full Marks</i> : +4, If <b>ONLY</b> the bubble corresponding to the correct answer is darkened.  <i>Zero Marks</i> : 0, In all other cases.</p>	

Q.49 Consider the  $3 \times 3$  matrices  $A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$  and  $I = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ . Then the trace of the matrix  $(A^3 - 6I)$  is \_\_\_\_\_

Q.50 Let  $S$  be the area of the region in the first quadrant enclosed by the three lines  $x = 0, y = 0, x = 2$ , and the graph of the function

$$f(x) = \begin{cases} 2 & \text{if } 0 \leq x \leq 1, \\ \frac{2}{x^2} & \text{if } x > 1. \end{cases}$$

Then the value of  $S$  is \_\_\_\_\_

Q.51 Let

$$f(x) = \int_{-1}^x t|t|dt.$$

Then the value of  $3f(2)$  is \_\_\_\_\_

Q.52 Out of a group of 6 girls and 4 boys, a team of three is formed at random. Let  $p$  be the probability that the team consists of 2 girls and 1 boy. Then the value of  $4p$  is \_\_\_\_\_

Q.53 While entering data consisting of 10 numbers, a person makes a mistake of entering a two-digit number in the reverse order. In doing so the mean of the data decreases by 1.8. Then the absolute value of the difference of the digits of the two-digit number is \_\_\_\_\_

Q.54 The number of values of  $\theta$  in the interval  $\left[0, \frac{\pi}{2}\right]$  satisfying the equation

$$\sin 2\theta - \cos 2\theta = 1 + \sin \theta - \cos \theta$$

is \_\_\_\_\_



Q.57

PARAGRAPH VIII

A fair coin is tossed three times. Let  $E_1$  be the event that exactly 2 heads appear,  $E_2$  be the event that exactly 1 head appears, and  $E_3$  be the event that at least 1 head appears.

The conditional probability  $P(E_1|E_3)$  is

(A)  $\frac{3}{8}$

(B)  $\frac{3}{7}$

(C)  $\frac{4}{7}$

(D)  $\frac{5}{8}$

Q.58

PARAGRAPH VIII

A fair coin is tossed three times. Let  $E_1$  be the event that exactly 2 heads appear,  $E_2$  be the event that exactly 1 head appears, and  $E_3$  be the event that at least 1 head appears.

The conditional probability  $P(E_2|E_3)$  is

(A)  $\frac{3}{7}$

(B)  $\frac{3}{8}$

(C)  $\frac{4}{7}$

(D)  $\frac{5}{8}$

Q.59

PARAGRAPH IX

Let  $L$  be the normal to the parabola  $y^2 = 16x$  at the point  $(1, 4)$  on the parabola.

The  $x$ -intercept of the line  $L$  is

(A)  $\frac{3}{2}$

(B) 9

(C) 7

(D)  $\frac{7}{2}$

Q.60

PARAGRAPH IX

Let  $L$  be the normal to the parabola  $y^2 = 16x$  at the point  $(1, 4)$  on the parabola.

The  $y$ -intercept of the line  $L$  is

(A)  $\frac{3}{2}$

(B) 9

(C) 7

(D)  $\frac{9}{2}$