Subject Part – 1: Physics
SECTION – I (32 Marks)

- This section contains **EIGHT (8)** questions.
- Each question has FOUR options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.
- For each question, darken the bubble on the OMR sheet corresponding to the correct option.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : $+4$ If **ONLY** the correct option is chosen;
Zero Marks : 0 If none of the options is chosen (i.e., the question is unanswered).
Negative Marks : -1 In all other cases.

Q.1 The physical quantity that has the same S.I. unit as that of torque is

(A) gravitational force.
(B) gravitational potential energy.
(C) rate of change of angular velocity
(D) linear momentum.

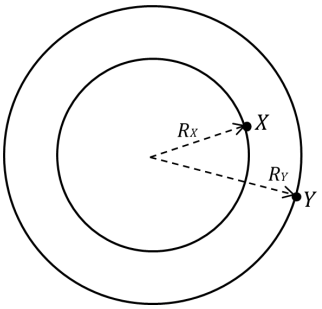
Q.2 α, β, and γ rays of equal energy are bombarded on a lead block. If r_α, r_β, and r_γ denote the penetration depths of α, β, and γ rays, respectively, the correct relationship is

(A) $r_\alpha > r_\beta = r_\gamma$ (B) $r_\beta > r_\alpha > r_\gamma$ (C) $r_\gamma > r_\alpha > r_\beta$ (D) $r_\gamma > r_\beta > r_\alpha$

Q.3 The magnetic field component of a plane monochromatic electromagnetic wave is expressed as $\vec{B} = -B_0\hat{y}sin(3 \times 10^7 z - 6 \times 10^{15} t)$. Here, $\hat{y}$ represents a unit vector along the $+y$-direction. The direction of polarization of the wave and refractive index of the medium in which the wave is travelling, are
[Take the speed of light in free space as $3 \times 10^8\ m\ sec^{-1}$, $\pm\,\hat{x}$ denote a unit vector along the $\pm x$ axis]

(A) $-\hat{x}$ and 1.5 (B) $+\hat{x}$ and 1.5 (C) $-\hat{x}$ and 1.0 (D) $+\hat{x}$ and 1.0

Q.4 Objects X and Y, initially at rest, start moving anticlockwise on concentric circles of radii R_X and R_Y $(R_Y > R_X)$ with constant angular accelerations of $\frac{\pi}{10}$ and $\frac{\pi}{5}\ rad\ sec^{-2}$, respectively. The figure below depicts positions of the objects X and Y at some later time t. If the initial distance between the objects X and Y is $R_Y - R_X$, then the distance between the two objects after $10\ sec$ is



(A) $R_Y - R_X$ (B) $R_Y + R_X$ (C) $\sqrt{(R_Y^2 + R_Y^2)}$ (D) $2R_X + R_y$

Q.5 Match the quantities in column I and column II, based on their dimensions.

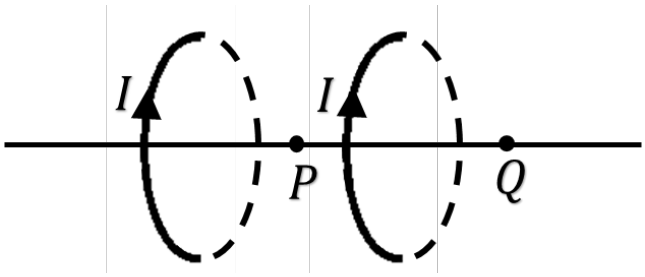
I	II
i. Electric field	p. Magnetic field × area
ii. Magnetic flux	q. Electric force/charge
iii. Charge	r. Current × area
iv. Magnetic dipole moment	s. Current × time

(A) i → p; ii → q; iii → r; iv → s
(B) i → q; ii → p; iii → s; iv → r
(C) i → r; ii → s; iii → p; iv → q
(D) i → q; ii → r; iii → s; iv → p

Q.6 An object of mass $1\ kg$, initially at rest, is dropped vertically down from a height of $45\ m$. The speed of the object, when it reaches the ground, will be _____ $m\ sec^{-1}$.
[Take acceleration due to gravity as $10\ m\ sec^{-2}$, and neglect air resistance]

(A) 30 (B) 25 (C) 45 (D) 20

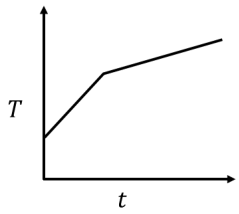
Q.7 Two circular loops of radius R, having a common axis passing through their centers, are kept at a distance $\frac{R}{4}$ from each other. Each loop carries a current I in the clockwise direction, as shown schematically in the figure below. On the axis, point P is between the two loops and point Q is outside. The correct statement for the magnitude of the magnetic field is



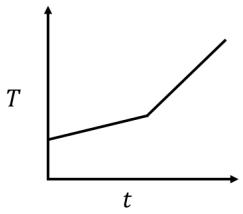
(A) magnetic field at P = magnetic field at Q.
(B) magnetic field at P > magnetic field at Q.
(C) magnetic field at P < magnetic field at Q.
(D) magnetic field at P = 0.

Q.8 A closed vessel contains only water in the liquid state. The vessel is supplied heat at a constant rate. Heating is continued even after all of the liquid is converted to vapor. The heat capacities of water in liquid and vapor states are constant. The heat capacity of water in the liquid state is more than that in the vapor state. The plot that best describes the variation of the temperature (T) of water with time (t) is

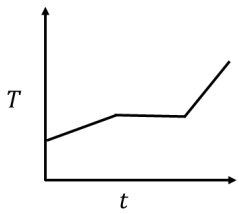
(A)



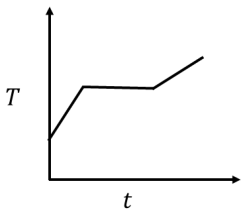
(B)



(C)



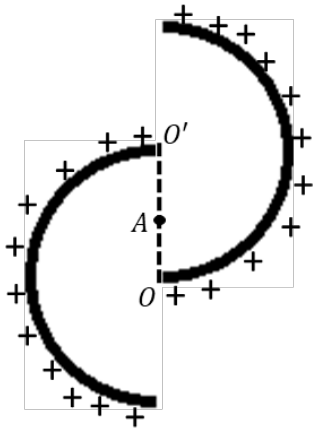
(D)



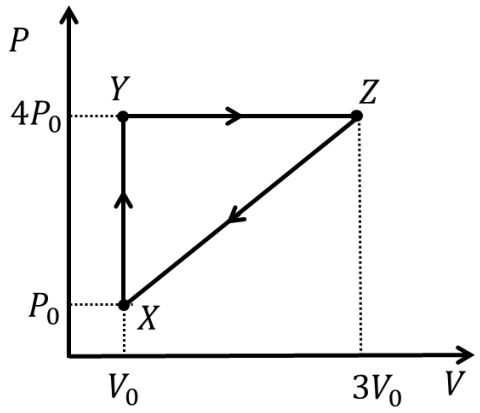
Subject Part – 1: Physics
SECTION – II (24 Marks)

- This section contains **Six (6)** questions.
- The answer to each question is a **SINGLE DIGIT NON-NEGATIVE INTEGER** ranging from 0 to 9, both inclusive
- For each question, darken the correct digit on your OMR Sheet. Do not write the answer on the Question paper.
- Answer to each question will be evaluated according to the following marking scheme:
  Full Marks : +4 If **ONLY** the correct digit is darkened;
  Zero Marks : 0 In all other cases.

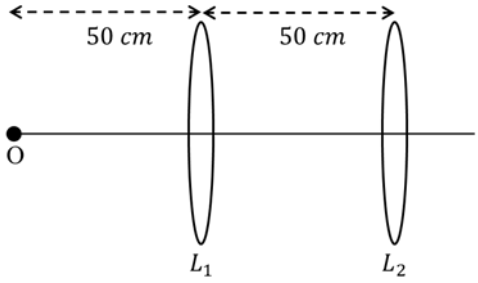
Q.9 Positive charges are uniformly distributed over two thin semi-circular arcs of radii R and centered at O and O', as shown in figure below. A is the mid-point of line OO'. The linear charge density on the arcs is λ and ϵ_0 is the permittivity of free space. If the magnitude of the electric field at the point A is $\alpha \frac{\lambda}{2\pi\epsilon_0}$, then value of α is _____.



Q.10 An ideal monoatomic gas undergoes a reversible, cyclic thermodynamic process $X \rightarrow Y \rightarrow Z \rightarrow X$ as shown on the pressure (P) versus volume (V) diagram below. The heat absorbed by the gas during this cyclic process is $\alpha P_0 V_0$. The value of α is _____ .

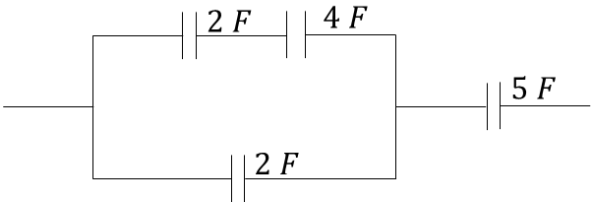


Q.11 A combination of two thin biconvex lenses L_1 and L_2 with focal lengths, $f_1 = 10\ cm$ and $f_2 = 20\ cm$, respectively, is shown in the figure below. A point object 'O' is placed $50\ cm$ away from L_1. The image of the point object is formed on the optic axis at $\frac{300}{n}\ cm$ from L_2 on its right side. The value of n is _____.



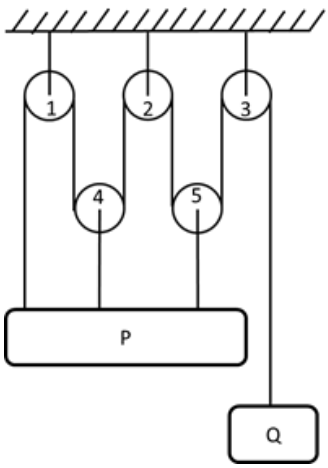
Q.12 In the photoelectric effect, the threshold frequency for a given metal is $4.53 \times 10^{14}\ Hz$. The work function of the metal is estimated to be $n \times 10^{-19}\ J$. The value of n is _____.
(rounded off to the nearest integer). [Take the value of Planck constant as $6.63 \times 10^{-34}\ J\ sec$]

Q.13 Consider the following combination of capacitors, as shown in the figure below:



The equivalent capacitance (in F) of the combination is _____ .

Q.14 The arrangement of pulleys shown below supports two blocks P and Q. The string and all the pulleys are ideal. Pulleys 1, 2, and 3 are fixed and pulleys 4 and 5 can move along the vertical direction. If block Q falls with an acceleration of $5\ m\ sec^{-2}$, then the acceleration of block P (in $m\ sec^{-2}$) in the upward direction is _____.



Subject Part – 1: Physics
SECTION – III (24 Marks)

- This section contains **Three (3)** paragraphs.
- Based on each paragraph, there are **TWO (2)** questions.
- The **FIRST** question is of multiple choice type having **FOUR OPTIONS** (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.
  - For this question, darken the bubble on the OMR sheet corresponding to the correct option.
  - Answer to each question will be evaluated according to the following marking scheme:
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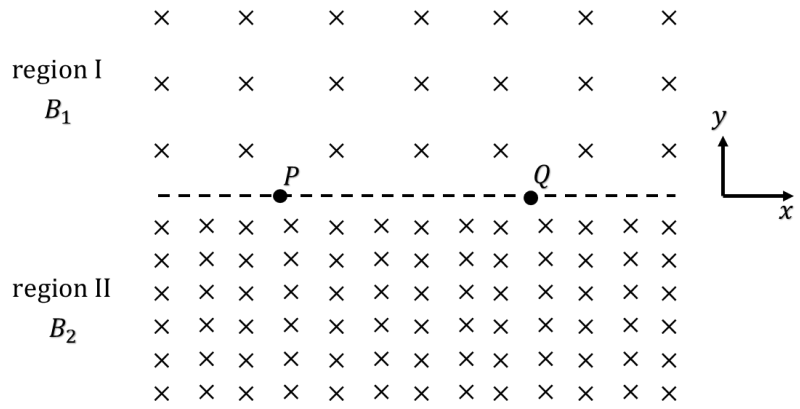
    Zero Marks : 0 In all other cases.
- The answer to the **SECOND** question is a NUMERICAL VALUE.
  If the calculated numerical value has more than ONE decimal place, truncate/round-off the value to ONE decimal place.
  - For this question, darken the correct digits on your OMR Sheet.
  - Answer to each question will be evaluated according to the following marking scheme
    Full Marks : +4 If **ONLY** the correct numerical value is darkened;
    Zero Marks : 0 In all other cases.

PARAGRAPH I

Two adjacent regions, I and II, having uniform magnetic fields B_1 and B_2, respectively, are shown in the figure below. The magnetic fields in the two regions are pointing in the $-z$ direction. A negatively charged particle starts moving from point P at the boundary between the two regions with initial velocity $v_o\hat{y}$. After undergoing a motion through regions I and II, it re-enters region I at point Q with velocity, $v_o\hat{y}$.

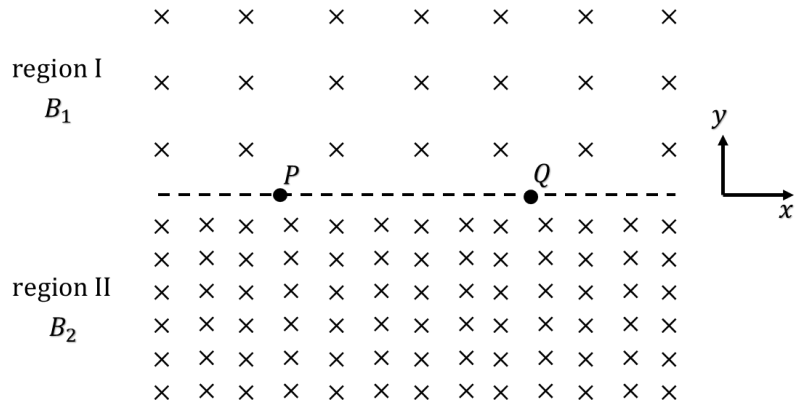


Q.15 Two different experiments are performed with particles, '1' and '2', having different masses m_1 and m_2, respectively, but same charge and initial velocity. The radius of the trajectory of particle 1 in region I is the same as the radius of the trajectory of particle 2 in region II. If $\frac{B_2}{B_1} = \frac{5}{4}$, then the value of $\frac{m_1}{m_2}$ is

(A) $\frac{4}{5}$ (B) $\frac{5}{4}$ (C) $\frac{2}{\sqrt{5}}$ (D) $\frac{\sqrt{5}}{2}$

PARAGRAPH I

Two adjacent regions, I and II, having uniform magnetic fields B_1 and B_2, respectively, are shown in the figure below. The magnetic fields in the two regions are pointing in the $-z$ direction. A negatively charged particle starts moving from point P at the boundary between the two regions with initial velocity $v_o\hat{y}$. After undergoing a motion through regions I and II, it re-enters region I at point Q with velocity, $v_o\hat{y}$.



Q.16 Let $\frac{B_2}{B_1} = \frac{5}{4}$ and R_1 and R_2 be the radii of the particle trajectories in regions I and II, respectively. The distance between P and Q is αR_2. The value of α is _____.

PARAGRAPH II

A point particle of mass, $m = 100\ g$ is suspended on a vertical mass-less spring. The rest length of the spring is L and its spring-constant is $k = 10\ Nm^{-1}$. After attaching the mass, the spring is stretched and came to a new equilibrium position. From equilibrium position, the particle is given a small displacement and released so that it executes a simple harmonic motion. It is given that the acceleration due to gravity is $10\ m\ sec^{-2}$.

Q.17 If the highest point of the oscillations is L from the point of suspension of the spring, the amplitude of the oscillations (in cm) is

(A) 2 (B) 4 (C) 10 (D) 20

PARAGRAPH II

A point particle of mass, $m = 100\ g$ is suspended on a vertical mass-less spring. The rest length of the spring is L and its spring-constant is $k = 10\ Nm^{-1}$. After attaching the mass, the spring is stretched and came to a new equilibrium position. From equilibrium position, the particle is given a small displacement and released so that it executes a simple harmonic motion. It is given that the acceleration due to gravity is $10\ m\ sec^{-2}$.

Q.18 The time period (in sec) of the oscillations is _____.

PARAGRAPH III

In photoelectric effect, the kinetic energy (K) of the ejected electrons (mass m) depends on the wavelengths (λ) of the incident light and the work function (ϕ) of the metal surface. To explain the effect, the light needs to be considered as photons of energy, $E = \frac{hc}{\lambda}$, where h is the Planck's constant and c is the speed of light in vacuum. Therefore, the threshold wavelength (λ_{th}) of the light is just enough to eject the electrons near the surface. Take electronic charge to be e.

Q.19 The fastest speed of the electrons is v for an incident light of wavelength λ. Upon changing the wavelength of the light to $\frac{\lambda}{2}$, the new maximum speed will be

(A) $\left(v^2 + \frac{2hc}{m\lambda}\right)^{1/2}$ (B) $\left(v^2 + \frac{hc}{m\lambda}\right)^{1/2}$ (C) $\left(v^2 + \frac{hc}{2m\lambda}\right)^{1/2}$ (D) $\left(2v^2 + \frac{hc}{m\lambda}\right)^{1/2}$

PARAGRAPH III

In photoelectric effect, the kinetic energy (K) of the ejected electrons (mass m) depends on the wavelengths (λ) of the incident light and the work function (ϕ) of the metal surface. To explain the effect, the light needs to be considered as photons of energy, $E = \frac{hc}{\lambda}$, where h is the Planck's constant and c is the speed of light in vacuum. Therefore, the threshold wavelength (λ_{th}) of the light is just enough to eject the electrons near the surface. Take electronic charge to be e.

Q.20 The light is shone on the metal surface for time T in which there are total N electrons ejected out from the isolated metal surface. A resultant equipotential metal surface with a net additional potential V gets developed. If $V = \frac{\phi}{2e}$, the threshold wavelength of the light for this metal surface decreases by a factor of α. The value of α is _____.

Subject Part – 2: Chemistry
SECTION – I (32 Marks)

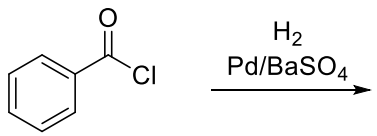
- This section contains **EIGHT (8)** questions.
- Each question has FOUR options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.
- For each question, darken the bubble on the OMR sheet corresponding to the correct option.
- Answer to each question will be evaluated according to the following marking scheme:

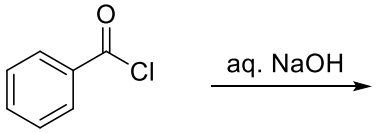
Full Marks : $+4$ If **ONLY** the correct option is chosen;
Zero Marks : 0 If none of the options is chosen (i.e., the question is unanswered).
Negative Marks : -1 In all other cases.

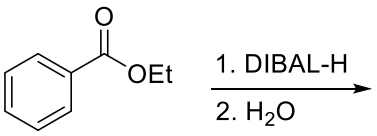
Q.21 The atomic orbital that has zero nodes is

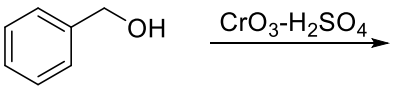
(A) $1s$ (B) $2s$ (C) $3s$ (D) $4s$

Q.22 The reactions that give benzaldehyde as the major product are

(i) 

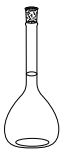
(ii) 

(iii) 

(iv) 

(A) (i) and (iii)
(B) (i) and (iv)
(C) (ii) and (iii)
(D) (iii) and (iv)

Q.23 The picture that represents a volumetric flask is

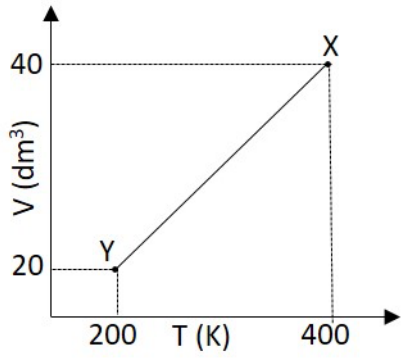
(A) 

(B) 

(C) 

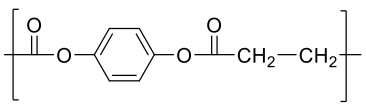
(D) 

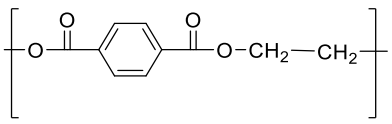
Q.24 One mole of an ideal gas undergoes a reversible process from $X \longrightarrow Y$. A plot of volume (V) versus temperature (T) for the process is given below. The work done (in L atm) during the process X to Y is
[Given: R = 0.082 L atm mol^{-1} K^{-1}]

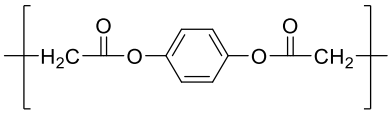


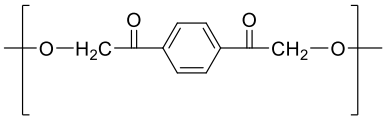
(A) 40 (B) 8.2 (C) 16.4 (D) 32.8

Q.25 The repeating unit of Terylene is

(A) 

(B) 

(C) 

(D) 

Q.26 The most ionic compound among the following is

(A) LiCl (B) CsCl (C) $BeCl_2$ (D) $MgCl_2$

Q.27 Match the compounds given in **Column-I** with their corresponding shapes given in **Column-II** and choose the correct option

Column-I	**Column-II**
Compound	**Shape**
i. SiF_4	P. T-Shaped
ii. XeF_4	Q. Pyramidal
iii. BrF_3	R. Square Planar
iv. NH_3	S. Tetrahedral

(A) i → S; ii → R; iii → Q; iv → P
(B) i → R; ii → S; iii → Q; iv → P
(C) i → R; ii → S; iii → P; iv → Q
(D) i → S; ii → R; iii → P; iv → Q

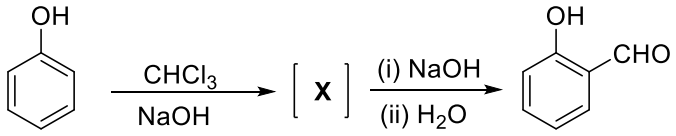
Q.28 The radioactive decay that results in the reduction of mass of a nucleus by 4 and its atomic number by 2 is

(A) α decay (B) β^+ decay (C) β^- decay (D) γ decay

Subject Part – 2: Chemistry
SECTION – II (24 Marks)

- This section contains **Six (6)** questions.
- The answer to each question is a **SINGLE DIGIT NON-NEGATIVE INTEGER** ranging from 0 to 9, both inclusive
- For each question, darken the correct digit on your OMR Sheet. Do not write the answer on the Question paper.
- Answer to each question will be evaluated according to the following marking scheme:
  Full Marks : +4 If **ONLY** the correct digit is darkened;
  Zero Marks : 0 In all other cases.

Q.29 In the following reaction, the number of chlorine atom(s) present in the intermediate **X** is _____.



Q.30 The pH of the solution when 75 mL of 0.02 M HCl solution is mixed with 25 mL of 0.02 M NaOH is _____.

Q.31 The total number of Lewis acid(s) among the following compounds is _____.

BBr_3, NH_3, H_2O, $AlCl_3$, HCl

Q.32 The enthalpy change (ΔH in kJ mol^{-1}) for the reaction $2A + B \rightarrow A_2B$ is _____.
[Given:
$2A \rightarrow A_2$ $\Delta H_1 = -55$ kJ mol^{-1}
$A_2B \rightarrow A_2 + B$ $\Delta H_2 = -60$ kJ mol^{-1}]

Q.33 The volume of SO_2 gas (in L) required to be added to 1 L of CO_2 gas to get an equimolar mixture of both the gases is _____.

[Assume gases are ideal and both the gases are at the same T and P]

Q.34 The sum of the lone pair(s) of electrons on the central atom of XeF_6 and XeF_4 is _____.

Subject Part – 2: Chemistry
SECTION – III (24 Marks)

- This section contains **Three (3)** paragraphs.
- Based on each paragraph, there are **TWO (2)** questions.
- The **FIRST** question is of multiple choice type having **FOUR OPTIONS** (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.
    - For this question, darken the bubble on the OMR sheet corresponding to the correct option.
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  If the calculated numerical value has more than ONE decimal place, truncate/round-off the value to ONE decimal place.
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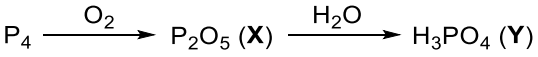
    - Answer to each question will be evaluated according to the following marking scheme
      Full Marks : +4 If **ONLY** the correct numerical value is darkened;
      Zero Marks : 0 In all other cases.

PARAGRAPH IV

Elemental phosphorus reacts with an excess of molecular oxygen to form an oxide **X**. Addition of water to **X** leads to the formation of **Y**. The reaction sequence is given below.



Q.35 Choose the correct statement from the following.

(A) The geometry around the central atom of **Y** is tetrahedral.

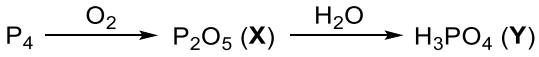
(B) Only two OH groups are present in each molecule of **Y**.

(C) **Y** is basic in nature.

(D) The phosphorus atoms of **X** and **Y** have one lone pair of electrons each.

PARAGRAPH IV

Elemental phosphorus reacts with an excess of molecular oxygen to form an oxide **X**. Addition of water to **X** leads to the formation of **Y**. The reaction sequence is given below.



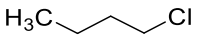
Q.36 The formal oxidation state of phosphorus in **X** is $+n$. The value of n is ____.

PARAGRAPH V

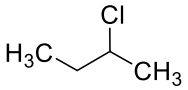
Alkyl chlorides participate in a variety of transformations, such as nucleophilic substitution, elimination, radical reactions, etc.

Q.37 The compound that shows the highest rate of reactivity towards S_N1 reaction is

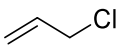
(A)



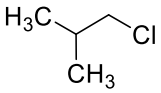
(B)



(C)



(D)



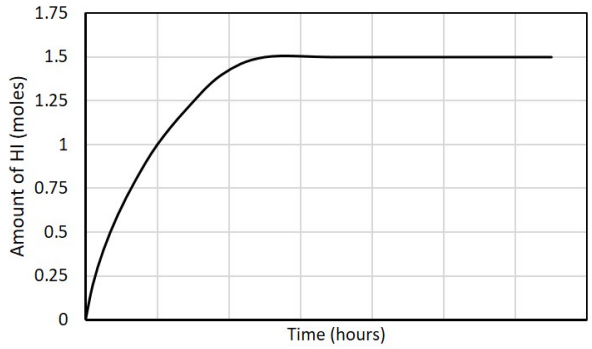
PARAGRAPH V

Alkyl chlorides participate in a variety of transformations, such as nucleophilic substitution, elimination, radical reactions, etc.

Q.38 The reaction of *tert*-butyl chloride in the presence of sodium metal in dry ether gives a major product **P**. The total number of methyl groups (CH_3) present in **P** is ____.

PARAGRAPH VI

When 1 mole of H_2 is mixed with 1 mole of I_2 in a one liter closed vessel in presence of a catalyst, the amount of HI formed as a function of time is given below

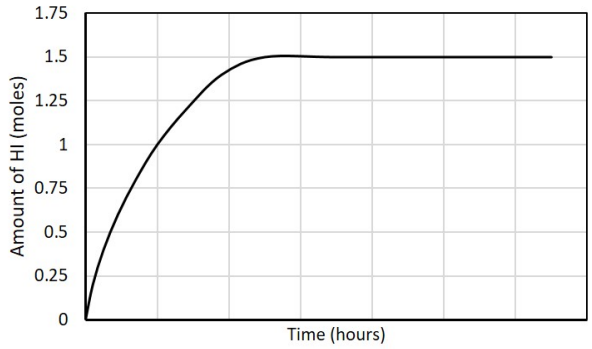


Q.39 Equilibrium constant, K_c, for the reaction, $H_2(g) + I_2(g) \rightleftharpoons 2\,HI(g)$, is

(A) 9 (B) 18 (C) 72 (D) 36

PARAGRAPH VI

When 1 mole of H_2 is mixed with 1 mole of I_2 in a one liter closed vessel in presence of a catalyst, the amount of HI formed as a function of time is given below



Q.40 If 0.4 mole of HI is formed between 2 h and 4 h, the rate of reaction (in mol L^{-1} h^{-1}) of $H_2(g) + I_2(g) \rightleftharpoons 2\,HI(g)$ in this interval is ____.

Rough work

Subject Part – 3: Mathematics
SECTION – I (32 Marks)

- This section contains **EIGHT (8)** questions.
- Each question has FOUR options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.
- For each question, darken the bubble on the OMR sheet corresponding to the correct option.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : $+4$ If **ONLY** the correct option is chosen;
Zero Marks : 0 If none of the options is chosen (i.e., the question is unanswered).
Negative Marks : -1 In all other cases.

Q.41 The limit

$$\lim_{x \to 0} \frac{e^{-x^2} - 1 - x|x|}{|x|}$$

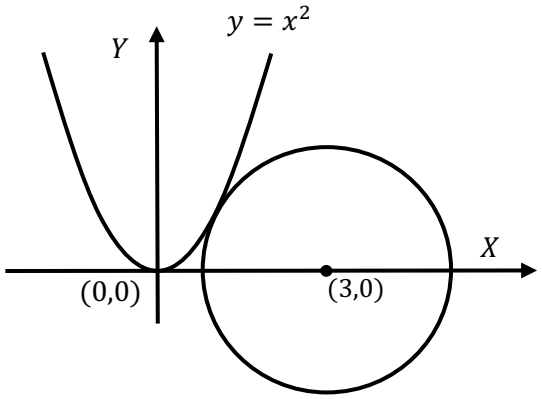
(A) is 0 (B) is $\frac{1}{2}$ (C) is $-\frac{1}{2}$ (D) does not exist

Q.42 Match the following tables

Table-1		Table-2
i.	The plane $x + 2y - 3z + 1 = 0$ passes through the point	P. $(7, 8, 8)$
ii.	For $\vec{a} = \hat{\imath} + 2\hat{\jmath} + 3\hat{k}$, $\vec{b} = 2\hat{\imath} + 3\hat{\jmath} + 5\hat{k}$ and $M = (2, 4, 9)$, the foot of the perpendicular drawn from M to the line $\vec{r} = \vec{a} + \lambda\vec{b}$ is	Q. $(-1, -1, -1)$
iii.	The centroid of the triangle with vertices at $(1, 1, 3), (12, 14, 18)$ and $(2, 9, 24)$ is	R. $(5, 8, 15)$
iv.	The point of intersection of the line $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and the plane $5x + 6y - 9z + 2 = 0$ is	S. $(3, 5, 8)$

(A) i → P, ii → Q, iii → R, iv → S
(B) i → Q, ii → S, iii → P, iv → R
(C) i → P, ii → S, iii → R, iv → Q
(D) i → S, ii → R, iii → P, iv → Q

Q.43 Which one of the following is the equation of the circle centered at $(3, 0)$ which touches (tangentially) the parabola $y = x^2$ exactly at one point?



(A) $(x-3)^2 + y^2 = 5$ (B) $(x-3)^2 + y^2 = 4$
(C) $(x-3)^2 + y^2 = 6$ (D) $(x-3)^2 + y^2 = 7$

Q.44 Let PQR and UVW be right-angled triangles that are similar. Given that the sides of PQR are in arithmetic progression and the perimeter of UVW, in cm, is 36. Then the area of UVW, in sq. cm, is

(A) 24 (B) 36 (C) 48 (D) 54

Q.45 The area bounded by the curves

$$y = x|x| \text{ and } y = 1 - 3x^2$$

is equal to

(A) $\frac{\sqrt{3}+1}{3}$ (B) $\frac{\sqrt{3}-1}{3}$ (C) $\frac{\sqrt{2}-1}{3}$ (D) $\frac{\sqrt{2}+1}{3}$

Q.46 For $0 < \theta < \frac{\pi}{2}$, if $\tan\theta - 2\cot\theta = 0$, then $2\,\text{cosec}^2\theta + \sec^2\theta$ is

(A) 4 (B) 6 (C) 8 (D) 9

Q.47 For real numbers a and b, consider the following system of linear equations in variables x, y, z:

$$\begin{aligned} x + y + z &= 0 \\ 2ax + by + 3z &= 0 \\ ay + bz &= 0 \end{aligned}$$

Then the subset

$$\{(a, a-b) : \text{the above system has infinitely many solutions}\}$$

of the Cartesian plane represents

(A) a circle
(B) an ellipse but not a circle
(C) a parabola
(D) a pair of straight lines

Q.48 Let X and Y be two events such that $P(X) = \frac{1}{4}$, $P(X|Y) = \frac{1}{2}$ and $P(Y|X) = \frac{1}{4}$. Which one of the following statements is true?

(A) X and Y are independent events

(B) X and Y are exhaustive events

(C) $P(X), P(Y)$ and $P(X \cap Y)$ are in geometric progression

(D) Y is twice as likely to occur as X

Subject Part – 3: Mathematics
SECTION – II (24 Marks)

- This section contains **Six (6)** questions.
- The answer to each question is a **SINGLE DIGIT NON-NEGATIVE INTEGER** ranging from 0 to 9, both inclusive
- For each question, darken the correct digit on your OMR Sheet. Do not write the answer on the Question paper.
- Answer to each question will be evaluated according to the following marking scheme:
  Full Marks : +4 If **ONLY** the correct digit is darkened;
  Zero Marks : 0 In all other cases.

Q.49 For 70 pens bought in different stores, it is found that

$$\sum_{i=1}^{70}(x_i - 7) = 35\,,$$

where x_i denotes the price of the i-th pen. If the mean price of another 30 pens is found to be 12.5 , then the mean price of all 100 pens is _____ .

Q.50 Consider the function f on the real line defined by

$$f(x) = a^2 + b^2 - \sqrt{2}ab\,(\sin x + \cos x\,),$$

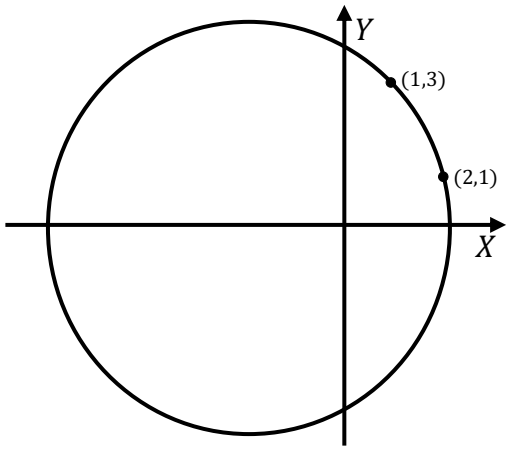
where a and b are positive constants. If the maximum value of f is 16 and the minimum value of f is 0, then the value of $a + 3b$ is _____.

Q.51 Let $2, a, b\ (a > b)$ be the three roots of the polynomial $x^3 - 10x^2 + Nx - 30$, where N is a real number. Then the value of $(a-b)^5 - N$ is _____.

Q.52 Denote by $[x]$, the greatest integer less than or equal to x. The value of the following integral is _____.

$$\pi^2 \int_{-1}^{2} ([x] - |x|)\cos \pi x\, dx$$

Q.53 Let C be the circle having its center on the x-axis such that the points $(2,1)$ and $(1,3)$ lie on C. If r is the radius of C, then the value of $\frac{10r}{\sqrt{85}}$ is ____.



Q.54 For $a, b, c, d \in (-\infty, \infty)$, consider the function

$$f(x) = \begin{cases} -1\,, & x < 0 \\ ax^3 + b\,x^2 + cx + d\,, & 0 \le x \le 1 \\ 1\,, & x > 1 \end{cases}$$

If f is differentiable for all $x \in (-\infty, \infty)$, then the value of $b - a + c + d$ is _____

Subject Part – 3: Mathematics
SECTION – III (24 Marks)

- This section contains **Three (3)** paragraphs.
- Based on each paragraph, there are **TWO (2)** questions.
- The **FIRST** question is of multiple choice type having **FOUR OPTIONS** (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.
  - For this question, darken the bubble on the OMR sheet corresponding to the correct option.
  - Answer to each question will be evaluated according to the following marking scheme:

    Full Marks : $+4$ If **ONLY** the correct option is chosen;
    Zero Marks : 0 In all other cases.

- The answer to the **SECOND** question is a NUMERICAL VALUE.
  If the calculated numerical value has more than ONE decimal place, truncate/round-off the value to ONE decimal place.
  - For this question, darken the correct digits on your OMR Sheet.
  - Answer to each question will be evaluated according to the following marking scheme

    Full Marks : $+4$ If **ONLY** the correct numerical value is darkened;
    Zero Marks : 0 In all other cases.

PARAGRAPH VII

Consider the lines given below:

$$L_1: \frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4},$$

$$L_2: \frac{x-2}{4} = \frac{y-3}{5} = \frac{z-4}{6},$$

and

$$L_3: \frac{x-3}{3} = \frac{y-4}{8} = \frac{z-5}{15}.$$

Q.55 Which one of the following statements is true?

(A) Only L_1 and L_2 are coplanar
(B) Only L_2 and L_3 are coplanar
(C) Only L_1 and L_3 are coplanar
(D) L_1, L_2 and L_3 all are coplanar

PARAGRAPH VII

Consider the lines given below:

$$L_1: \frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4},$$

$$L_2: \frac{x-2}{4} = \frac{y-3}{5} = \frac{z-4}{6},$$

and

$$L_3: \frac{x-3}{3} = \frac{y-4}{8} = \frac{z-5}{15}.$$

Q.56 If the distance between L_2 and L_3 is d unit, then the value of $\frac{\sqrt{2782}}{5}d$ is _____.

PARAGRAPH VIII

A competition is conducted in a city for students of n schools $S_1, S_2, \ldots, S_n$, where $n \geq 4$. Let the probability that a student from the school S_i wins the competition is p_i. Assume that

$$\sum_{i=1}^{n} p_i = 1.$$

Let $p_1 = \frac{1}{10}, p_2 = \frac{1}{12}$ and $p_3 = \frac{1}{15}$.

Q.57 If a student who is NOT from S_1 wins the competition, then what is the probability that the student is from S_2 ?

(A) $\frac{1}{54}$ (B) $\frac{11}{54}$ (C) $\frac{5}{54}$ (D) $\frac{7}{54}$

PARAGRAPH VIII

A competition is conducted in a city for students of n schools $S_1, S_2, \ldots, S_n$, where $n \geq 4$. Let the probability that a student from the school S_i wins the competition is p_i. Assume that

$$\sum_{i=1}^{n} p_i = 1.$$

Let $p_1 = \frac{1}{10}, p_2 = \frac{1}{12}$ and $p_3 = \frac{1}{15}$.

Q.58 Suppose a student wins the competition. If p is the probability that the student is either from the school S_2 or from the school S_3, then the value of $10p$ is ____.

PARAGRAPH IX

Consider the function f defined on the real line by

$$f(x) = \begin{cases} \tan^{-1}\left(\dfrac{2x-3}{x+3}\right), & x \neq -3 \\ -\dfrac{\pi}{2}, & x = -3 \end{cases}$$

Here $\tan^{-1}\theta$ denotes the principal value branch of the inverse of the function $\tan\theta$.

Q.59 Which one of the following statements is true?

(A) f is increasing in $(-\infty, -3)$
(B) f is decreasing in $(-\infty, 0)$ and increasing in $(0, \infty)$
(C) f is increasing in $(-\infty, -3)$ and decreasing in $(-3, \infty)$
(D) f is decreasing in $(-\infty, \infty)$

PARAGRAPH IX

Consider the function f defined on the real line by

$$f(x) = \begin{cases} \tan^{-1}\left(\dfrac{2x-3}{x+3}\right), & x \neq -3 \\ -\dfrac{\pi}{2}, & x = -3 \end{cases}$$

Here $\tan^{-1}\theta$ denotes the principal value branch of the inverse of the function $\tan\theta$.

Q.60 Let f^{-1} denote the inverse of the function f. Then the value of $\frac{1}{2}(f^{-1})'\left(\frac{\pi}{4}\right)$ is ____.

END OF THE QUESTION PAPER

Rough Work

Rough Work

Rough Work