

**Subject Part – 1: Physics**  
**SECTION – I (32 Marks)**

- This section contains **EIGHT (8)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.
- For each question, darken the bubble on the OMR sheet corresponding to the correct option.
- Answer to each question will be evaluated according to the following marking scheme:
 

*Full Marks* : +4 If **ONLY** the correct option is chosen;  
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*Negative Marks* : -1 In all other cases.

Q.1  $\alpha$ ,  $\beta$ , and  $\gamma$  rays of equal energy are bombarded on a lead block. If  $r_\alpha$ ,  $r_\beta$ , and  $r_\gamma$  denote the penetration depths of  $\alpha$ ,  $\beta$ , and  $\gamma$  rays, respectively, the correct relationship is

(A)  $r_\alpha > r_\beta = r_\gamma$       (B)  $r_\beta > r_\alpha > r_\gamma$       (C)  $r_\gamma > r_\alpha > r_\beta$       (D)  $r_\gamma > r_\beta > r_\alpha$

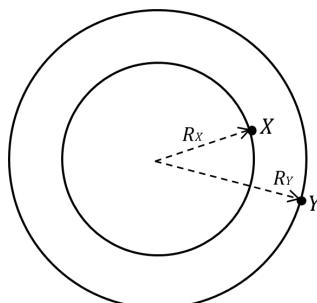
Q.2 The magnetic field component of a plane monochromatic electromagnetic wave is expressed as  $\vec{B} = -B_0 \hat{y} \sin(3 \times 10^7 z - 6 \times 10^{15} t)$ . Here,  $\hat{y}$  represents a unit vector along the  $+y$ -direction. The direction of polarization of the wave and refractive index of the medium in which the wave is travelling, are  
 [Take the speed of light in free space as  $3 \times 10^8 \text{ m sec}^{-1}$ ,  $\pm \hat{x}$  denote a unit vector along the  $\pm x$  axis]

(A)  $-\hat{x}$  and 1.5      (B)  $+\hat{x}$  and 1.5      (C)  $-\hat{x}$  and 1.0      (D)  $+\hat{x}$  and 1.0

Q.3 An object of mass 1 kg, initially at rest, is dropped vertically down from a height of 45 m. The speed of the object, when it reaches the ground, will be \_\_\_\_\_  $\text{m sec}^{-1}$ .  
 [Take acceleration due to gravity as  $10 \text{ m sec}^{-2}$ , and neglect air resistance]

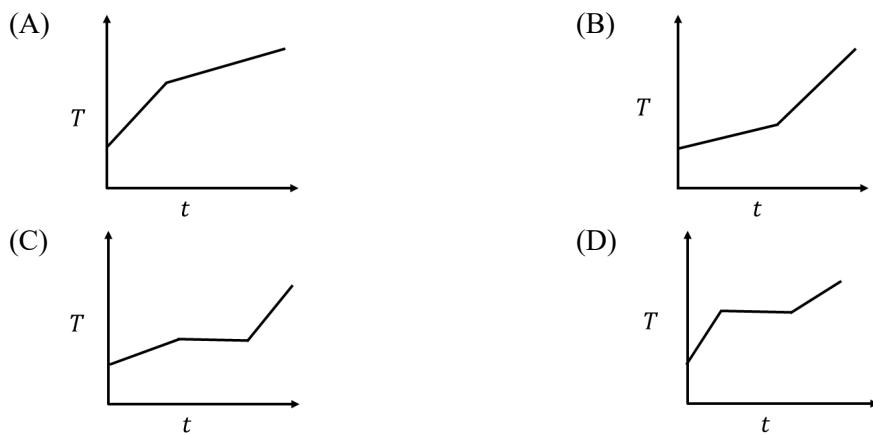
(A) 30      (B) 25      (C) 45      (D) 20

Q.4 Objects  $X$  and  $Y$ , initially at rest, start moving anticlockwise on concentric circles of radii  $R_X$  and  $R_Y$  ( $R_Y > R_X$ ) with constant angular accelerations of  $\frac{\pi}{10}$  and  $\frac{\pi}{5} \text{ rad sec}^{-2}$ , respectively. The figure below depicts positions of the objects  $X$  and  $Y$  at some later time  $t$ . If the initial distance between the objects  $X$  and  $Y$  is  $R_Y - R_X$ , then the distance between the two objects after 10 sec is



(A)  $R_Y - R_X$       (B)  $R_Y + R_X$       (C)  $\sqrt{(R_Y^2 + R_X^2)}$       (D)  $2R_X + R_Y$

Q.5 A closed vessel contains only water in the liquid state. The vessel is supplied heat at a constant rate. Heating is continued even after all of the liquid is converted to vapor. The heat capacities of water in liquid and vapor states are constant. The heat capacity of water in the liquid state is more than that in the vapor state. The plot that best describes the variation of the temperature ( $T$ ) of water with time ( $t$ ) is

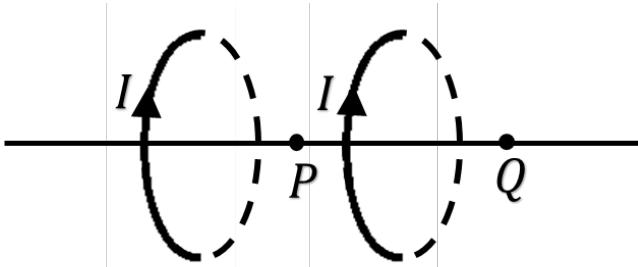


Q.6 Match the quantities in column I and column II, based on their dimensions.

I	II
i. Electric field	p. Magnetic field $\times$ area
ii. Magnetic flux	q. Electric force/charge
iii. Charge	r. Current $\times$ area
iv. Magnetic dipole moment	s. Current $\times$ time

(A) i  $\rightarrow$  p; ii  $\rightarrow$  q; iii  $\rightarrow$  r; iv  $\rightarrow$  s      (B) i  $\rightarrow$  q; ii  $\rightarrow$  p; iii  $\rightarrow$  s; iv  $\rightarrow$  r  
 (C) i  $\rightarrow$  r; ii  $\rightarrow$  s; iii  $\rightarrow$  p; iv  $\rightarrow$  q      (D) i  $\rightarrow$  q; ii  $\rightarrow$  r; iii  $\rightarrow$  s; iv  $\rightarrow$  p

Q.7 Two circular loops of radius  $R$ , having a common axis passing through their centers, are kept at a distance  $\frac{R}{4}$  from each other. Each loop carries a current  $I$  in the clockwise direction, as shown schematically in the figure below. On the axis, point P is between the two loops and point Q is outside. The correct statement for the magnitude of the magnetic field is



(A) magnetic field at P = magnetic field at Q.  
 (B) magnetic field at P > magnetic field at Q.  
 (C) magnetic field at P < magnetic field at Q.  
 (D) magnetic field at P = 0.

Q.8 The physical quantity that has the same S.I. unit as that of torque is

- (A) gravitational force.
- (B) gravitational potential energy.
- (C) rate of change of angular velocity.
- (D) linear momentum.

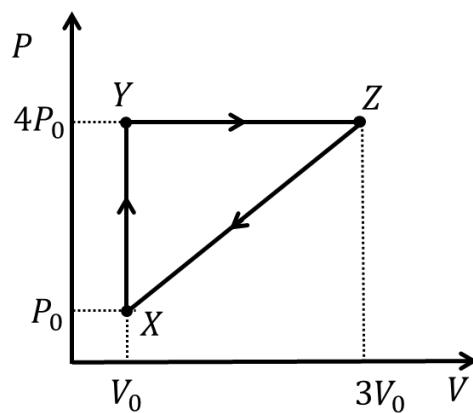
**Subject Part – 1: Physics**  
**SECTION – II (24 Marks)**

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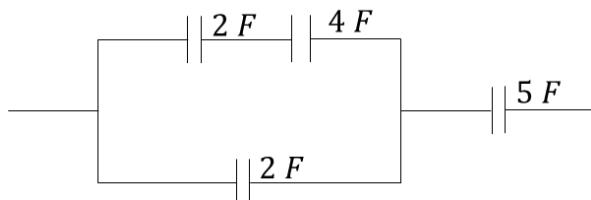
**Full Marks** : +4 If **ONLY** the correct digit is darkened;  
**Zero Marks** : 0 In all other cases.

Q.9 In the photoelectric effect, the threshold frequency for a given metal is  $4.53 \times 10^{14} \text{ Hz}$ . The work function of the metal is estimated to be  $n \times 10^{-19} \text{ J}$ . The value of  $n$  is \_\_\_\_\_. (rounded off to the nearest integer). [Take the value of Planck constant as  $6.63 \times 10^{-34} \text{ J sec}$ ]

Q.10 An ideal monoatomic gas undergoes a reversible, cyclic thermodynamic process  $X \rightarrow Y \rightarrow Z \rightarrow X$  as shown on the pressure ( $P$ ) versus volume ( $V$ ) diagram below. The heat absorbed by the gas during this cyclic process is  $\alpha P_0 V_0$ . The value of  $\alpha$  is \_\_\_\_\_.

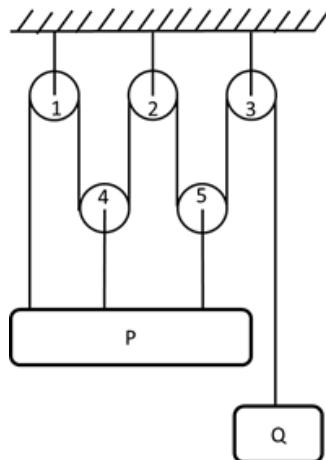


Q.11 Consider the following combination of capacitors, as shown in the figure below:

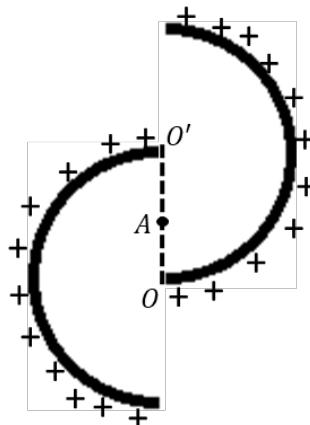


The equivalent capacitance (in  $F$ ) of the combination is \_\_\_\_\_.

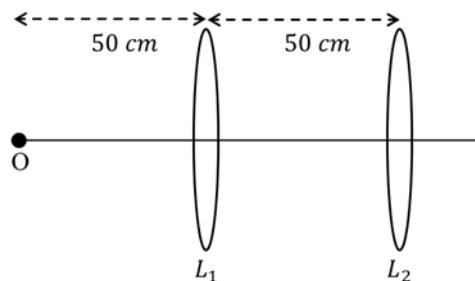
Q.12 The arrangement of pulleys shown below supports two blocks P and Q. The string and all the pulleys are ideal. Pulleys 1, 2, and 3 are fixed and pulleys 4 and 5 can move along the vertical direction. If block Q falls with an acceleration of  $5 \text{ m sec}^{-2}$ , then the acceleration of block P (in  $\text{m sec}^{-2}$ ) in the upward direction is \_\_\_\_\_.



Q.13 Positive charges are uniformly distributed over two thin semi-circular arcs of radii  $R$  and centered at  $O$  and  $O'$ , as shown in figure below. A is the mid-point of line  $OO'$ . The linear charge density on the arcs is  $\lambda$  and  $\epsilon_0$  is the permittivity of free space. If the magnitude of the electric field at the point A is  $\alpha \frac{\lambda}{2\pi\epsilon_0}$ , then value of  $\alpha$  is \_\_\_\_\_.



Q.14 A combination of two thin biconvex lenses  $L_1$  and  $L_2$  with focal lengths,  $f_1 = 10 \text{ cm}$  and  $f_2 = 20 \text{ cm}$ , respectively, is shown in the figure below. A point object 'O' is placed  $50 \text{ cm}$  away from  $L_1$ . The image of the point object is formed on the optic axis at  $\frac{300}{n} \text{ cm}$  from  $L_2$  on its right side. The value of  $n$  is \_\_\_\_\_.



## **Subject Part – 1: Physics**

### **SECTION – III (24 Marks)**

- This section contains **Three (3)** paragraphs.
- Based on each paragraph, there are **TWO (2)** questions.
- The **FIRST** question is of multiple choice type having **FOUR OPTIONS** (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.
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## PARAGRAPH I

A point particle of mass,  $m = 100\text{ g}$  is suspended on a vertical mass-less spring. The rest length of the spring is  $L$  and its spring-constant is  $k = 10\text{ Nm}^{-1}$ . After attaching the mass, the spring is stretched and came to a new equilibrium position. From equilibrium position, the particle is given a small displacement and released so that it executes a simple harmonic motion. It is given that the acceleration due to gravity is  $10\text{ m sec}^{-2}$ .

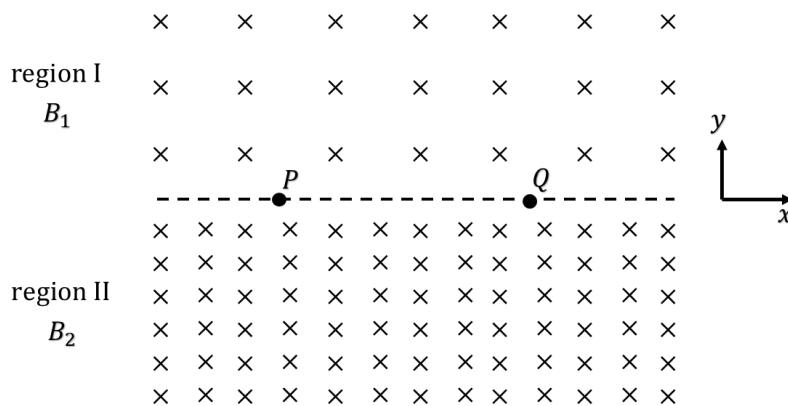
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Q.16 The time period (in sec) of the oscillations is \_\_\_\_.

## PARAGRAPH II

Two adjacent regions, I and II, having uniform magnetic fields  $B_1$  and  $B_2$ , respectively, are shown in the figure below. The magnetic fields in the two regions are pointing in the  $-z$  direction. A negatively charged particle starts moving from point  $P$  at the boundary between the two regions with initial velocity  $v_0\hat{y}$ . After undergoing a motion through regions I and II, it re-enters region I at point  $Q$  with velocity  $v_0\hat{y}$ .

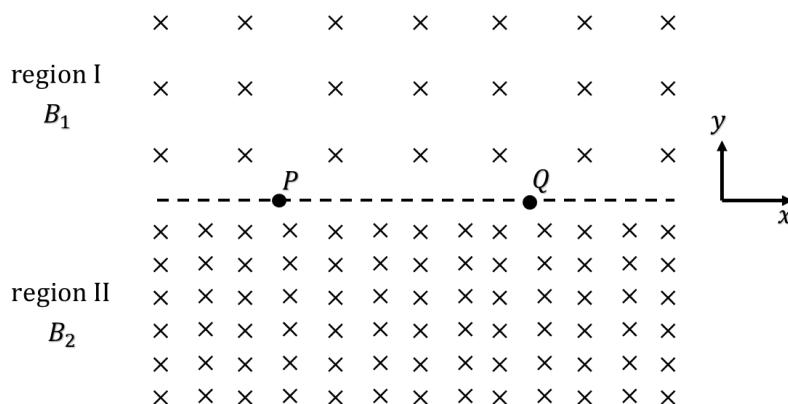


Q.17 Two different experiments are performed with particles, '1' and '2', having different masses  $m_1$  and  $m_2$ , respectively, but same charge and initial velocity. The radius of the trajectory of particle 1 in region I is the same as the radius of the trajectory of particle 2 in region II. If  $\frac{B_2}{B_1} = \frac{5}{4}$ , then the value of  $\frac{m_1}{m_2}$  is

(A)  $\frac{4}{5}$       (B)  $\frac{5}{4}$       (C)  $\frac{2}{\sqrt{5}}$       (D)  $\frac{\sqrt{5}}{2}$

## PARAGRAPH II

Two adjacent regions, I and II, having uniform magnetic fields  $B_1$  and  $B_2$ , respectively, are shown in the figure below. The magnetic fields in the two regions are pointing in the  $-z$  direction. A negatively charged particle starts moving from point  $P$  at the boundary between the two regions with initial velocity  $v_0\hat{y}$ . After undergoing a motion through regions I and II, it re-enters region I at point  $Q$  with velocity  $v_0\hat{y}$ .



Q.18 Let  $\frac{B_2}{B_1} = \frac{5}{4}$  and  $R_1$  and  $R_2$  be the radii of the particle trajectories in regions I and II, respectively. The distance between  $P$  and  $Q$  is  $\alpha R_2$ . The value of  $\alpha$  is \_\_\_\_\_.  
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## PARAGRAPH III

In photoelectric effect, the kinetic energy ( $K$ ) of the ejected electrons (mass  $m$ ) depends on the wavelengths ( $\lambda$ ) of the incident light and the work function ( $\phi$ ) of the metal surface. To explain the effect, the light needs to be considered as photons of energy,  $E = \frac{hc}{\lambda}$ , where  $h$  is the Planck's constant and  $c$  is the speed of light in vacuum. Therefore, the threshold wavelength ( $\lambda_{th}$ ) of the light is just enough to eject the electrons near the surface. Take electronic charge to be  $e$ .

Q.19 The fastest speed of the electrons is  $v$  for an incident light of wavelength  $\lambda$ . Upon changing the wavelength of the light to  $\frac{\lambda}{2}$ , the new maximum speed will be

(A)  $\left(v^2 + \frac{2hc}{m\lambda}\right)^{1/2}$       (B)  $\left(v^2 + \frac{hc}{m\lambda}\right)^{1/2}$       (C)  $\left(v^2 + \frac{hc}{2m\lambda}\right)^{1/2}$       (D)  $\left(2v^2 + \frac{hc}{m\lambda}\right)^{1/2}$

## PARAGRAPH III

In photoelectric effect, the kinetic energy ( $K$ ) of the ejected electrons (mass  $m$ ) depends on the wavelengths ( $\lambda$ ) of the incident light and the work function ( $\phi$ ) of the metal surface. To explain the effect, the light needs to be considered as photons of energy,  $E = \frac{hc}{\lambda}$ , where  $h$  is the Planck's constant and  $c$  is the speed of light in vacuum. Therefore, the threshold wavelength ( $\lambda_{th}$ ) of the light is just enough to eject the electrons near the surface. Take electronic charge to be  $e$ .

Q.20 The light is shone on the metal surface for time  $T$  in which there are total  $N$  electrons ejected out from the isolated metal surface. A resultant equipotential metal surface with a net additional potential  $V$  gets developed. If  $V = \frac{\phi}{2e}$ , the threshold wavelength of the light for this metal surface decreases by a factor of  $\alpha$ . The value of  $\alpha$  is \_\_\_\_\_.

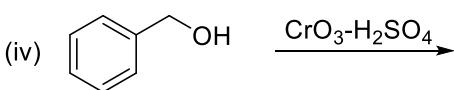
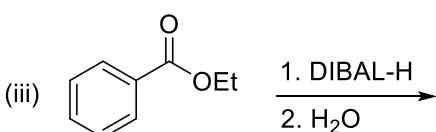
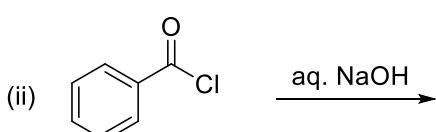
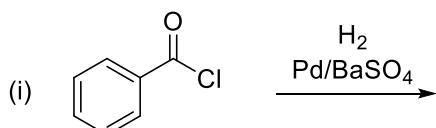
## **Subject Part – 2: Chemistry**

### **SECTION – I (32 Marks)**

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Q.21 The most ionic compound among the following is

Q.22 The reactions that give benzaldehyde as the major product are



(A) (i) and (iii) (B) (i) and (iv)  
(C) (ii) and (iii) (D) (iii) and (iv)

Q.23 The picture that represents a volumetric flask is

(A)



(B)



(C)

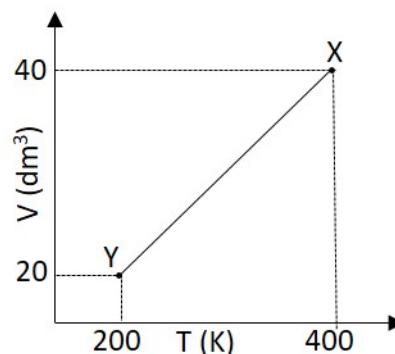


(D)



Q.24 One mole of an ideal gas undergoes a reversible process from  $X \rightarrow Y$ . A plot of volume ( $V$ ) versus temperature ( $T$ ) for the process is given below. The work done (in L atm) during the process  $X$  to  $Y$  is

[Given:  $R = 0.082 \text{ L atm mol}^{-1} \text{ K}^{-1}$ ]



(A) 40

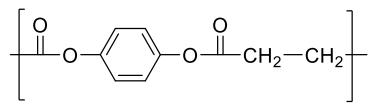
(B) 8.2

(C) 16.4

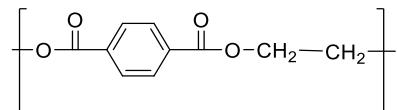
(D) 32.8

Q.25 The repeating unit of Terylene is

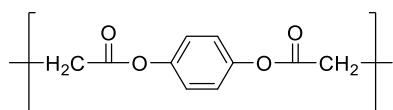
(A)



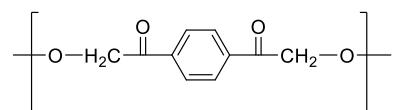
(B)



(C)



(D)



Q.26 The atomic orbital that has zero nodes is

(A) 1s (B) 2s (C) 3s (D) 4s

Q.27 The radioactive decay that results in the reduction of mass of a nucleus by 4 and its atomic number by 2 is

(A)  $\alpha$  decay (B)  $\beta^+$  decay (C)  $\beta^-$  decay (D)  $\gamma$  decay

Q.28 Match the compounds given in **Column-I** with their corresponding shapes given in **Column-II** and choose the correct option

Column-I	Column-II
Compound	Shape
i. $\text{SiF}_4$	P. T-Shaped
ii. $\text{XeF}_4$	Q. Pyramidal
iii. $\text{BrF}_3$	R. Square Planar
iv. $\text{NH}_3$	S. Tetrahedral

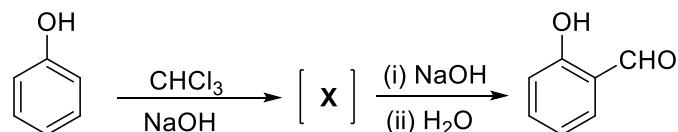
(A) i  $\rightarrow$  S; ii  $\rightarrow$  R; iii  $\rightarrow$  Q; iv  $\rightarrow$  P  
(B) i  $\rightarrow$  R; ii  $\rightarrow$  S; iii  $\rightarrow$  Q; iv  $\rightarrow$  P  
(C) i  $\rightarrow$  R; ii  $\rightarrow$  S; iii  $\rightarrow$  P; iv  $\rightarrow$  Q  
(D) i  $\rightarrow$  S; ii  $\rightarrow$  R; iii  $\rightarrow$  P; iv  $\rightarrow$  Q

**Subject Part – 2: Chemistry**  
**SECTION – II (24 Marks)**

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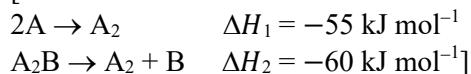
Q.29 In the following reaction, the number of chlorine atom(s) present in the intermediate **X** is \_\_\_\_\_.



Q.30 The sum of the lone pair(s) of electrons on the central atom of  $\text{XeF}_6$  and  $\text{XeF}_4$  is \_\_\_\_\_.

Q.31 The pH of the solution when 75 mL of 0.02 M HCl solution is mixed with 25 mL of 0.02 M NaOH is \_\_\_\_\_.

Q.32 The enthalpy change ( $\Delta H$  in  $\text{kJ mol}^{-1}$ ) for the reaction  $2\text{A} + \text{B} \rightarrow \text{A}_2\text{B}$  is \_\_\_\_\_.  
 [Given:



Q.33 The total number of Lewis acid(s) among the following compounds is \_\_\_\_\_.



Q.34 The volume of  $\text{SO}_2$  gas (in L) required to be added to 1 L of  $\text{CO}_2$  gas to get an equimolar mixture of both the gases is \_\_\_\_\_.

[Assume gases are ideal and both the gases are at the same  $T$  and  $P$ ]

**Subject Part – 2: Chemistry**  
**SECTION – III (24 Marks)**

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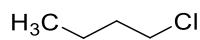
<i>Full Marks</i>	:	+4	If <b>ONLY</b> the correct numerical value is darkened;
<i>Zero Marks</i>	:	0	In all other cases.

**PARAGRAPH IV**

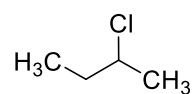
Alkyl chlorides participate in a variety of transformations, such as nucleophilic substitution, elimination, radical reactions, etc.

Q.35 The compound that shows the highest rate of reactivity towards  $S_N1$  reaction is

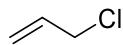
(A)



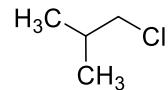
(B)



(C)



(D)



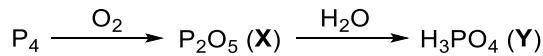
**PARAGRAPH IV**

Alkyl chlorides participate in a variety of transformations, such as nucleophilic substitution, elimination, radical reactions, etc.

Q.36 The reaction of *tert*-butyl chloride in the presence of sodium metal in dry ether gives a major product **P**. The total number of methyl groups ( $\text{CH}_3$ ) present in **P** is \_\_\_\_.

**PARAGRAPH V**

Elemental phosphorus reacts with an excess of molecular oxygen to form an oxide **X**. Addition of water to **X** leads to the formation of **Y**. The reaction sequence is given below.

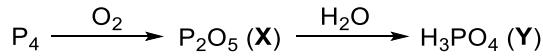


Q.37 Choose the correct statement from the following.

- (A) The geometry around the central atom of **Y** is tetrahedral.
- (B) Only two OH groups are present in each molecule of **Y**.
- (C) **Y** is basic in nature.
- (D) The phosphorus atoms of **X** and **Y** have one lone pair of electrons each.

**PARAGRAPH V**

Elemental phosphorus reacts with an excess of molecular oxygen to form an oxide **X**. Addition of water to **X** leads to the formation of **Y**. The reaction sequence is given below.

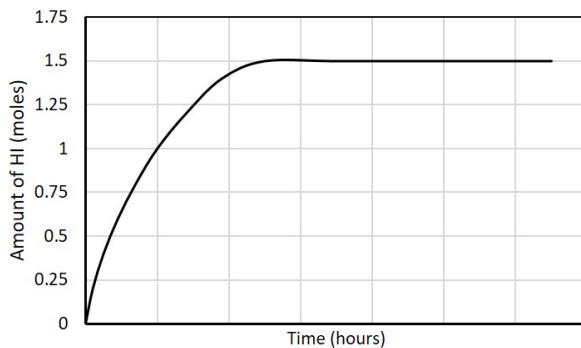


Q.38 The formal oxidation state of phosphorus in **X** is  $+n$ . The value of  $n$  is \_\_\_\_.

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## PARAGRAPH VI

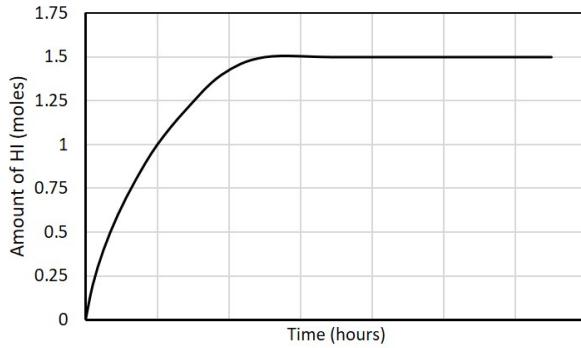
When 1 mole of  $\text{H}_2$  is mixed with 1 mole of  $\text{I}_2$  in a one liter closed vessel in presence of a catalyst, the amount of  $\text{HI}$  formed as a function of time is given below



Q.39 Equilibrium constant,  $K_c$ , for the reaction,  $H_2(g) + I_2(g) \rightleftharpoons 2 HI(g)$ , is

## **PARAGRAPH VI**

When 1 mole of  $\text{H}_2$  is mixed with 1 mole of  $\text{I}_2$  in a one liter closed vessel in presence of a catalyst, the amount of  $\text{HI}$  formed as a function of time is given below



Q.40 If 0.4 mole of HI is formed between 2 h and 4 h, the rate of reaction (in  $\text{mol L}^{-1} \text{ h}^{-1}$ ) of  $\text{H}_2(g) + \text{I}_2(g) \rightleftharpoons 2 \text{HI}(g)$  in this interval is \_\_\_\_.

Rough work

**Subject Part – 3: Mathematics**  
**SECTION – I (32 Marks)**

- This section contains **EIGHT (8)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.
- For each question, darken the bubble on the OMR sheet corresponding to the correct option.
- Answer to each question will be evaluated according to the following marking scheme:
 

*Full Marks* : +4 If **ONLY** the correct option is chosen;  
*Zero Marks* : 0 If none of the options is chosen (i.e., the question is unanswered).  
*Negative Marks* : -1 In all other cases.

Q.41 Let  $X$  and  $Y$  be two events such that  $P(X) = \frac{1}{4}$ ,  $P(X|Y) = \frac{1}{2}$  and  $P(Y|X) = \frac{1}{4}$ . Which one of the following statements is true?

- (A)  $X$  and  $Y$  are independent events
- (B)  $X$  and  $Y$  are exhaustive events
- (C)  $P(X)$ ,  $P(Y)$  and  $P(X \cap Y)$  are in geometric progression
- (D)  $Y$  is twice as likely to occur as  $X$

Q.42 For  $0 < \theta < \frac{\pi}{2}$ , if  $\tan \theta - 2 \cot \theta = 0$ , then  $2 \operatorname{cosec}^2 \theta + \sec^2 \theta$  is

- (A) 4
- (B) 6
- (C) 8
- (D) 9

Q.43 For real numbers  $a$  and  $b$ , consider the following system of linear equations in variables  $x, y, z$ :

$$\begin{aligned} x + y + z &= 0 \\ 2ax + by + 3z &= 0 \\ ay + bz &= 0 \end{aligned}$$

Then the subset

$$\{(a, a - b) : \text{the above system has infinitely many solutions}\}$$

of the Cartesian plane represents

(A) a circle	(B) an ellipse but not a circle
(C) a parabola	(D) a pair of straight lines

Q.44 The area bounded by the curves

$$y = x|x| \text{ and } y = 1 - 3x^2$$

is equal to

(A)  $\frac{\sqrt{3}+1}{3}$

(B)  $\frac{\sqrt{3}-1}{3}$

(C)  $\frac{\sqrt{2}-1}{3}$

(D)  $\frac{\sqrt{2}+1}{3}$

Q.45 Match the following tables

Table-1	Table-2
i. The plane $x + 2y - 3z + 1 = 0$ passes through the point	P. (7, 8, 8)
ii. For $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$ , $\vec{b} = 2\hat{i} + 3\hat{j} + 5\hat{k}$ and $M = (2, 4, 9)$ , the foot of the perpendicular drawn from $M$ to the line $\vec{r} = \vec{a} + \lambda\vec{b}$ is	Q. (-1, -1, -1)
iii. The centroid of the triangle with vertices at (1, 1, 3), (12, 14, 18) and (2, 9, 24) is	R. (5, 8, 15)
iv. The point of intersection of the line $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and the plane $5x + 6y - 9z + 2 = 0$ is	S. (3, 5, 8)

(A) i  $\rightarrow$  P, ii  $\rightarrow$  Q, iii  $\rightarrow$  R, iv  $\rightarrow$  S

(B) i  $\rightarrow$  Q, ii  $\rightarrow$  S, iii  $\rightarrow$  P, iv  $\rightarrow$  R

(C) i  $\rightarrow$  P, ii  $\rightarrow$  S, iii  $\rightarrow$  R, iv  $\rightarrow$  Q

(D) i  $\rightarrow$  S, ii  $\rightarrow$  R, iii  $\rightarrow$  P, iv  $\rightarrow$  Q

Q.46 The limit

$$\lim_{x \rightarrow 0} \frac{e^{-x^2} - 1 - x|x|}{|x|}$$

(A) is 0

(B) is  $\frac{1}{2}$

(C) is  $-\frac{1}{2}$

(D) does not exist

Q.47 Let  $PQR$  and  $UVW$  be right-angled triangles that are similar. Given that the sides of  $PQR$  are in arithmetic progression and the perimeter of  $UVW$ , in cm, is 36. Then the area of  $UVW$ , in sq. cm, is

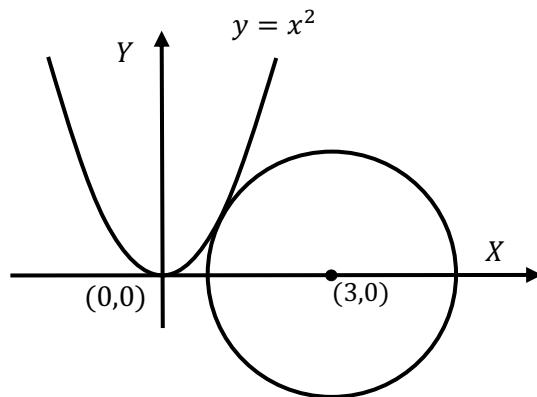
(A) 24

(B) 36

(C) 48

(D) 54

Q.48 Which one of the following is the equation of the circle centered at  $(3, 0)$  which touches (tangentially) the parabola  $y = x^2$  exactly at one point?



(A)  $(x - 3)^2 + y^2 = 5$       (B)  $(x - 3)^2 + y^2 = 4$   
(C)  $(x - 3)^2 + y^2 = 6$       (D)  $(x - 3)^2 + y^2 = 7$

**Subject Part – 3: Mathematics**  
**SECTION – II (24 Marks)**

- This section contains **Six (6)** questions.
- The answer to each question is a **SINGLE DIGIT NON-NEGATIVE INTEGER** ranging from 0 to 9, both inclusive
- For each question, darken the correct digit on your OMR Sheet. Do not write the answer on the Question paper.
- Answer to each question will be evaluated according to the following marking scheme:

*Full Marks* : +4 If **ONLY** the correct digit is darkened;  
*Zero Marks* : 0 In all other cases.

Q.49 Consider the function  $f$  on the real line defined by

$$f(x) = a^2 + b^2 - \sqrt{2}ab (\sin x + \cos x),$$

where  $a$  and  $b$  are positive constants. If the maximum value of  $f$  is 16 and the minimum value of  $f$  is 0, then the value of  $a + 3b$  is \_\_\_\_.

Q.50 For  $a, b, c, d \in (-\infty, \infty)$ , consider the function

$$f(x) = \begin{cases} -1, & x < 0 \\ ax^3 + b x^2 + cx + d, & 0 \leq x \leq 1 \\ 1, & x > 1 \end{cases}$$

If  $f$  is differentiable for all  $x \in (-\infty, \infty)$ , then the value of  $b - a + c + d$  is \_\_\_\_.

Q.51 Denote by  $[x]$ , the greatest integer less than or equal to  $x$ . The value of the following integral is \_\_\_\_.

$$\pi^2 \int_{-1}^2 ([x] - |x|) \cos \pi x \, dx$$

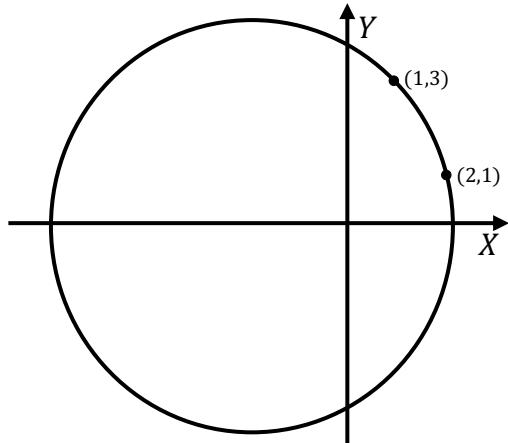
Q.52 Let  $2, a, b$  ( $a > b$ ) be the three roots of the polynomial  $x^3 - 10x^2 + Nx - 30$ , where  $N$  is a real number. Then the value of  $(a - b)^5 - N$  is \_\_\_\_.

Q.53 For 70 pens bought in different stores, it is found that

$$\sum_{i=1}^{70} (x_i - 7) = 35,$$

where  $x_i$  denotes the price of the  $i$ -th pen. If the mean price of another 30 pens is found to be 12.5, then the mean price of all 100 pens is \_\_\_\_\_.

Q.54 Let  $C$  be the circle having its center on the  $x$ -axis such that the points  $(2, 1)$  and  $(1, 3)$  lie on  $C$ . If  $r$  is the radius of  $C$ , then the value of  $\frac{10r}{\sqrt{85}}$  is \_\_\_\_\_.



**Subject Part – 3: Mathematics**  
**SECTION – III (24 Marks)**

- This section contains **Three (3)** paragraphs.
- Based on each paragraph, there are **TWO (2)** questions.
- The **FIRST** question is of multiple choice type having **FOUR OPTIONS** (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.
  - For this question, darken the bubble on the OMR sheet corresponding to the correct option.
  - Answer to each question will be evaluated according to the following marking scheme:  
 Full Marks : +4 If **ONLY** the correct option is chosen;  
 Zero Marks : 0 In all other cases.
- The answer to the **SECOND** question is a **NUMERICAL VALUE**.  
 If the calculated numerical value has more than **ONE** decimal place, truncate/round-off the value to **ONE** decimal place.
  - For this question, darken the correct digits on your OMR Sheet.
  - Answer to each question will be evaluated according to the following marking scheme:  
 Full Marks : +4 If **ONLY** the correct numerical value is darkened;  
 Zero Marks : 0 In all other cases.

**PARAGRAPH VII**

Consider the lines given below:

$$L_1: \frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4},$$

$$L_2: \frac{x-2}{4} = \frac{y-3}{5} = \frac{z-4}{6},$$

and

$$L_3: \frac{x-3}{3} = \frac{y-4}{8} = \frac{z-5}{15}.$$

Q.55 Which one of the following statements is true?

- (A) Only  $L_1$  and  $L_2$  are coplanar
- (B) Only  $L_2$  and  $L_3$  are coplanar
- (C) Only  $L_1$  and  $L_3$  are coplanar
- (D)  $L_1$ ,  $L_2$  and  $L_3$  all are coplanar

### PARAGRAPH VII

Consider the lines given below:

$$L_1: \frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4},$$

$$L_2: \frac{x-2}{4} = \frac{y-3}{5} = \frac{z-4}{6},$$

and

$$L_3: \frac{x-3}{3} = \frac{y-4}{8} = \frac{z-5}{15}.$$

Q.56 If the distance between  $L_2$  and  $L_3$  is  $d$  unit, then the value of  $\frac{\sqrt{2782}}{5}d$  is \_\_\_\_\_.

### PARAGRAPH VIII

Consider the function  $f$  defined on the real line by

$$f(x) = \begin{cases} \tan^{-1}\left(\frac{2x-3}{x+3}\right), & x \neq -3 \\ -\frac{\pi}{2}, & x = -3 \end{cases}$$

Here  $\tan^{-1} \theta$  denotes the principal value branch of the inverse of the function  $\tan \theta$ .

Q.57 Which one of the following statements is true?

- (A)  $f$  is increasing in  $(-\infty, -3)$
- (B)  $f$  is decreasing in  $(-\infty, 0)$  and increasing in  $(0, \infty)$
- (C)  $f$  is increasing in  $(-\infty, -3)$  and decreasing in  $(-3, \infty)$
- (D)  $f$  is decreasing in  $(-\infty, \infty)$

### PARAGRAPH VIII

Consider the function  $f$  defined on the real line by

$$f(x) = \begin{cases} \tan^{-1}\left(\frac{2x-3}{x+3}\right), & x \neq -3 \\ -\frac{\pi}{2}, & x = -3 \end{cases}$$

Here  $\tan^{-1} \theta$  denotes the principal value branch of the inverse of the function  $\tan \theta$ .

Q.58 Let  $f^{-1}$  denote the inverse of the function  $f$ . Then the value of  $\frac{1}{2}(f^{-1})'\left(\frac{\pi}{4}\right)$  is \_\_\_\_\_.

### PARAGRAPH IX

A competition is conducted in a city for students of  $n$  schools  $S_1, S_2, \dots, S_n$ , where  $n \geq 4$ . Let the probability that a student from the school  $S_i$  wins the competition is  $p_i$ . Assume that

$$\sum_{i=1}^n p_i = 1.$$

Let  $p_1 = \frac{1}{10}$ ,  $p_2 = \frac{1}{12}$  and  $p_3 = \frac{1}{15}$ .

Q.59 If a student who is NOT from  $S_1$  wins the competition, then what is the probability that the student is from  $S_2$  ?

(A)  $\frac{1}{54}$

(B)  $\frac{11}{54}$

(C)  $\frac{5}{54}$

(D)  $\frac{7}{54}$

### PARAGRAPH IX

A competition is conducted in a city for students of  $n$  schools  $S_1, S_2, \dots, S_n$ , where  $n \geq 4$ . Let the probability that a student from the school  $S_i$  wins the competition is  $p_i$ . Assume that

$$\sum_{i=1}^n p_i = 1.$$

Let  $p_1 = \frac{1}{10}$ ,  $p_2 = \frac{1}{12}$  and  $p_3 = \frac{1}{15}$ .

Q.60 Suppose a student wins the competition. If  $p$  is the probability that the student is either from the school  $S_2$  or from the school  $S_3$ , then the value of  $10p$  is \_\_\_\_.

### END OF THE QUESTION PAPER

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Rough Work

Rough Work

Rough Work