SECTION 1 Multiple Choice Question (MCQ)

- This section contains **TEN (10)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.
- For each question, darken the bubble corresponding to the correct answer.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : +3 If **ONLY** the bubble corresponding to the correct option is darkened.
Zero Marks : 0 If none of the bubble is darkened (i.e., the question is unanswered).
Negative Marks : −0.75 In all other cases.

Q.1 For a real number x, let $[x]$ denote the greatest integer less than or equal to x. Let $f: \mathbb{R} \to \mathbb{R}$ be the function defined by

$$f(x) = \begin{cases} \dfrac{([x^2] + [x^2 - 1]) \sin^3 x}{x^3} & \text{if } x \neq 0, \\ k & \text{if } x = 0. \end{cases}$$

Then the value of k for which f is continuous at 0 is

(A) 2 (B) -1 (C) 0 (D) 1

Q.2 Let z_1 and z_2 be complex numbers such that $i(z_1 - z_2)$ and $z_1 + z_2$ are both real numbers, where $i^2 = -1$. Which one of the following statements is true?

(A) $z_1 = \overline{z_2}$ (B) $z_1 = z_2$ (C) $z_1 = i\,\overline{z_2}$ (D) $z_1 = -\overline{z_2}$

Q.3 A heap of sand is in the form of a cone whose height is 9 meters and it contains $432\,\pi$ cubic meters of sand. The minimum area, in square meters, of the canvas required to cover the heap is

(A) $12\sqrt{63}\,\pi$ (B) $324\,\pi$ (C) $180\,\pi$ (D) $192\,\pi$

Q.4 The number of one-one (injective) functions from the set $A = \{-1, 0, 1\}$ to the set $B = \{1, 2, 3, 4\}$ is

(A) 12 (B) 81 (C) 64 (D) 24

Q.5 The value of θ in the interval $\left[0, \frac{\pi}{2}\right]$ satisfying $1 + \cos\theta + \cos^2\theta + \cos^3\theta + \cdots = 4 + 2\sqrt{3}$ is

(A) $\frac{\pi}{2}$ (B) $\frac{\pi}{3}$ (C) $\frac{\pi}{4}$ (D) $\frac{\pi}{6}$

Q.6 The number of possible 3×3 matrices with entries from the set $\{0, 1\}$ is

(A) 8 (B) 18 (C) 512 (D) 81

Q.7 Let $f : \mathbb{R} \to \mathbb{R}$ be a two times differentiable function such that $f''(x) \geq 0$ for every $x \in \mathbb{R}$ and $f'(0) > 0$, where f' and f'' denote the first and the second order derivatives of f, respectively. If $g(x) = f(e^x)$, then

(A) $g''(x) > 0$ for all $x \in \mathbb{R}$.

(B) $g''(x) = 0$ for some $x \in \mathbb{R}$.

(C) $g''(x) < 0$ for all $x \in \mathbb{R}$.

(D) There exist x_1 and x_2 in $\mathbb{R}$ such that $g''(x_1) < 0$ and $g''(x_2) > 0$.

Q.8 For which one of the following values of p, does the following inequality hold for all $x \geq 0$?

$$(1 + x)^p \leq 1 + x^p$$

(A) $p = \frac{3}{2}$ (B) $p = \frac{1}{2}$ (C) $p = 2$ (D) $p = 4$

Q.9 For which one of the following values of k, the equation

$$\cos\left(\frac{k\pi}{7}\right) = \cos\left(\frac{6\pi}{7}\right)$$

is satisfied?

(A) 9 (B) 5 (C) 8 (D) 4

Q.10 The value of the sum

$$\sum_{n=1}^{2024} \frac{1}{\sqrt{n+1} + \sqrt{n}}$$

is

(A) 24 (B) 45 (C) 20 (D) 44

SECTION 2 SDI

- This section contains **TEN (10)** questions.
- The answer to each question is a **SINGLE DIGIT NON-NEGATIVE INTEGER (SDI)**.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : +4 If **ONLY** the bubble corresponding to the correct answer is darkened.
Zero Marks : 0 In all other cases.

Q.11 The number of real solutions of the equation

$$\sqrt{x^2 - 1}\left(x^2 - 2x + \frac{3}{4}\right) = 0$$

is ______

Q.12 A boy of height 1.2 meters walks at a rate of 28 meters per minute away from a lamp which is 4 meters above the ground. If k meters per minute is the rate at which the length of the shadow of the boy is increasing, then the value of $\frac{k}{4}$ is ______

Q.13 The value of m for which the points $A = (-3,\ 1, -1),\ B = (1, m, 1)$, and $C = (-1, 2, 0)$ are collinear is ______

Q.14 For a complex number z, let $Re(z)$ denote its real part. If z_1 and z_2 are the non-real roots of the equation $(z-1)^3 - 1 = 0$, then the value of $Re(z_1) + Re(z_2)$ is ______

Q.15 The number of values of θ in the interval $\left[0, \frac{\pi}{2}\right]$ satisfying the equation

$$\sin 2\theta - \cos 2\theta = 1 + \sin\theta - \cos\theta$$

is ______

Q.16 While entering data consisting of 10 numbers, a person makes a mistake of entering a two-digit number in the reverse order. In doing so the mean of the data decreases by 1.8. Then the absolute value of the difference of the digits of the two-digit number is ______

Q.17 Out of a group of 6 girls and 4 boys, a team of three is formed at random. Let p be the probability that the team consists of 2 girls and 1 boy. Then the value of $4p$ is ______

Q.18 Let

$$f(x) = \int_{-1}^{x} t|t|dt.$$

Then the value of $3f(2)$ is ______

Q.19 Let S be the area of the region in the first quadrant enclosed by the three lines $x = 0, y = 0, x = 2$, and the graph of the function

$$f(x) = \begin{cases} 2 & \text{if } \; 0 \leq x \leq 1, \\ \dfrac{2}{x^2} & \text{if } \; x > 1. \end{cases}$$

Then the value of S is ______

Q.20 Consider the 3×3 matrices $A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$ and $I = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$. Then the trace of the matrix $(A^3 - 6I)$ is ______

SECTION 3: Paragraph based MCQ

- This section contains **FIVE (05)** paragraphs.
- Based on each paragraph, there are **TWO (02)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.
- If the numerical value has more than two decimal places, **truncate/round-off** the value to **TWO** decimal places.
- Answer to each question will be evaluated according to the following marking scheme:
  Full Marks : $+3$ If **ONLY** the bubble corresponding to the correct answer is darkened.
  Zero Marks : 0 If none of the bubble is darkened (i.e., the question is unanswered).
  Negative Marks : -0.75 In all other cases.

PARAGRAPH I

Consider the triangle ΔABC with sides AB, BC, CA in the ratio $1 : r : r^2$ for some $r > 0$. Suppose that the angle $\angle BCA = \alpha$ and the angle $\angle ABC = 2\alpha$.

Q.21 Which one of the following statements is true?

(A) $0 < \alpha < \frac{\pi}{6}$ (B) $\frac{\pi}{5} < \alpha < \frac{\pi}{3}$ (C) $\frac{\pi}{6} < \alpha < \frac{\pi}{5}$ (D) $\frac{\pi}{3} < \alpha < \frac{\pi}{2}$

PARAGRAPH I

Consider the triangle ΔABC with sides AB, BC, CA in the ratio $1 : r : r^2$ for some $r > 0$. Suppose that the angle $\angle BCA = \alpha$ and the angle $\angle ABC = 2\alpha$.

Q.22 Which one of the following statements is true?

(A) $r^2 = \cos\alpha$ (B) $r^2 = \sin\alpha$
(C) $r^2 = 2\cos\alpha$ (D) $r^2 = \cos 2\alpha$

PARAGRAPH II

A fair coin is tossed three times. Let E_1 be the event that exactly 2 heads appear, E_2 be the event that exactly 1 head appears, and E_3 be the event that at least 1 head appears.

Q.23 The conditional probability $P(E_2|E_3)$ is

(A) $\frac{3}{7}$ (B) $\frac{3}{8}$ (C) $\frac{4}{7}$ (D) $\frac{5}{8}$

PARAGRAPH II

A fair coin is tossed three times. Let E_1 be the event that exactly 2 heads appear, E_2 be the event that exactly 1 head appears, and E_3 be the event that at least 1 head appears.

Q.24 The conditional probability $P(E_1|E_3)$ is

(A) $\frac{3}{8}$ (B) $\frac{3}{7}$ (C) $\frac{4}{7}$ (D) $\frac{5}{8}$

PARAGRAPH III

Let L be the normal to the parabola $y^2 = 16x$ at the point $(1, 4)$ on the parabola.

Q.25 The y-intercept of the line L is

(A) $\frac{3}{2}$ (B) 9 (C) 7 (D) $\frac{9}{2}$

PARAGRAPH III

Let L be the normal to the parabola $y^2 = 16x$ at the point $(1, 4)$ on the parabola.

Q.26 The x-intercept of the line L is

(A) $\frac{3}{2}$ (B) 9 (C) 7 (D) $\frac{7}{2}$

PARAGRAPH IV

Let $\vec{u}$ and $\vec{v}$ be vectors such that $|\vec{u}| = 1,\ |\vec{v}| = 2|\vec{u}|,$ and $\vec{u}$ is perpendicular to $\vec{u} + \vec{v}$. Let $\vec{w}$ be a vector perpendicular to both $\vec{u}$ and $\vec{v}$ such that $\vec{w} \cdot (\vec{u} \times \vec{v}) = 6$.

Q.27 Which one of the following statements is true?

(A) $|\vec{w}| = 2\sqrt{3}$

(B) $\vec{w}$ is perpendicular to $\vec{u} \times \vec{v}$

(C) $\vec{w}$ is parallel to $\vec{u} + \vec{v}$

(D) $|\vec{w}| = \frac{\sqrt{3}}{2}$

PARAGRAPH IV

Let $\vec{u}$ and $\vec{v}$ be vectors such that $|\vec{u}| = 1$, $|\vec{v}| = 2|\vec{u}|$, and $\vec{u}$ is perpendicular to $\vec{u} + \vec{v}$. Let $\vec{w}$ be a vector perpendicular to both $\vec{u}$ and $\vec{v}$ such that $\vec{w} \cdot (\vec{u} \times \vec{v}) = 6$.

Q.28 Then the angle between $\vec{u}$ and $\vec{v}$ is

(A) $\frac{\pi}{3}$ (B) $\frac{2\pi}{3}$ (C) $\frac{3\pi}{4}$ (D) $\frac{5\pi}{6}$

PARAGRAPH V

Let $g(x) = x + \frac{1}{x}$ for $x \in \mathbb{R} \setminus \{0\}$, and let $Range(g)$ denote the range of the function g. Suppose $f: Range(g) \to \mathbb{R}$ is defined by

$$f\big(g(x)\big) = x^2 + \frac{1}{x^2} \cdot$$

Q.29 Which one of the following statements is true?

(A) g is one-one, but $f \circ g$ is not one-one.

(B) There exists $x \neq 0$ such that $f(g(x)) = 1$.

(C) For any real number $\alpha \geq 2$, there exists $x \in \mathbb{R}$ satisfying $f\big(g(x)\big) = \alpha$.

(D) $Range(f \circ g) = Range(g)$.

PARAGRAPH V

Let $g(x) = x + \frac{1}{x}$ for $x \in \mathbb{R} \setminus \{0\}$, and let $Range(g)$ denote the range of the function g. Suppose $f: Range(g) \to \mathbb{R}$ is defined by

$$f\big(g(x)\big) = x^2 + \frac{1}{x^2} \cdot$$

Q.30 The domain of the function f is

(A) $[-2,\ 2]$ (B) $(-\infty,\ -2] \cup [2,\ \infty)$
(C) $[2,\ \infty)$ (D) $(-\infty,\ -2]$